

Appropriate rankits to use for normal probability plots and Standard deviation probability plots

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Table of content

1	Introduction	3
2	Technical backgrounds.....	5
2.1	Symbols	5
2.2	Computation of B-rankits.....	6
2.3	Use of normal probability plots for checking the normality of a distribution	6
2.4	Use of normal probability plots for checking the outlying values	7
2.5	Use of normal probability plots for estimating mean values and standard deviations.....	8
2.6	Estimation of a standard deviation from repeated series	10
2.7	The binary search algorithm	11
2.8	The Monte-Carlo method	12
3	Laws of distribution of p_i	12
3.1	Introduction	12
3.2	Equations for p_i , mean value, centile and median values.....	13
3.3	Calculation of the mean values of p_i distributions.....	14
3.4	Calculation of the centiles and median values of p_i distributions	14
3.5	Use of p_i distributions in probability plots	15
3.6	Conclusions	18
4	Design of experiments	18
4.1	Introduction about appropriate P_i to use for probability plotting.....	18
4.2	Design of experiments for this study	18
5	Results and discussions.....	19
5.1	Determination the p_i distributions for $N = 2$ to 30	19

5.1.1	Results of the p_i distributions.....	19
5.1.2	Linear approximations of the median values in function of i and N	21
5.1.3	Polynomial approximations of the centile values in function of i and N	24
5.1.4	Conclusions concerning p_i distributions.....	27
5.2	z_i distributions for $N = 2$ to 30 for the Gaussian distribution.....	27
5.2.1	Introduction	27
5.2.2	Distributions of P_i (B-rankits) corresponding to Z_i (G-rankits), mean value of z_i	28
5.2.3	Determination of “a” with Monte-Carlo method	29
5.2.4	Comparison of methods to determine the ordinates at origin of the normal plotting curves (determination of the mean value).....	31
5.2.5	Comparison of methods to determine the slopes of the normal plotting curves (determination of the standard deviation).....	31
5.2.6	Intervals of confidence on z_i	34
5.2.7	Conclusions concerning z_i distributions for the Gaussian distribution	37
5.3	Determination of the z_{r_i} distributions for $N = 2$ to 30 for the distributions of standard deviation estimates of a Gaussian distributed population.....	38
5.3.1	Introduction	38
5.3.2	Distributions of p_i corresponding to z_{r_i}	39
5.3.3	Determination of “a” with Monte-Carlo method	40
5.3.4	Comparison of methods to determine abscissa corresponding to the ordinate $z_{r_i}=1$ of the standard deviation plotting curves (determination of a standard deviation)	44
5.3.5	Intervals of confidence on z_{r_i}	46
5.3.6	Conclusions concerning z_{r_i} distributions for the S-distribution of a Gaussian population	50
5.4	General conclusions	50
6	References	51

Annex:

Tables of detailed results that were obtained in this study.

Abstract:

Normal probability plots are usually used to check whether a distribution can be regarded as Gaussian, to visualise whether some figures are likely to be outliers and, using a linear regression, to estimate its mean value and its standard deviation. In the same way, “SD probability plots”, based on the distribution of standard deviation estimates, could be quite useful to reach similar goals: check whether a hypothesis of homoscedasticity can be accepted or not, visualise estimates that are likely to be outliers, and estimate the true underlying standard deviation. In practice, a change of variable is necessary to change the rank of each value into a corresponding cumulated probability and inverse Gaussian transformation to get a “rankit” to be used as ordinates for these plots. Equations in the form of $(i-a)/(N+1-2a)$ with $0 \leq a \leq 1$ are usually used to determine the adequate cumulated probabilities. As a matter of fact, at least for small values of N , the choice of the “ a ” value has an important impact on the conclusions that are drawn afterwards. This document:

- ✚ Discusses the grounds of these equations;
- ✚ Evaluates their adequacy for a series of situations and types of distribution laws;
- ✚ Proposes equations to determine “ a ” values as function of N , that provide better rankits than usually used and enable to estimate mean values and/or standard deviations without any bias for a series of situations;
- ✚ Proposes an accurate way to determine envelope curves of confidence for normal probability plots and probability plots of any distribution which cumulative function is known.

1 Introduction

Normal probability plots are frequently used to check whether a distribution can be regarded as Gaussian [1]. They also enable to visualise whether some data are likely to be outliers [2]. By using a linear regression, excluding as far as necessary some lower and upper values, they can also be used to estimate its mean value and its standard deviation.

Even if this not usually practised until now, “Standard deviation probability plots” could be in the same way quite useful to check homoscedasticity, outliers and produce robust estimations of a standard deviation in the same way than for normal probability plots.

In practice, a change of variable is necessary to change the rank of each value into a corresponding cumulated probability, that we call here “B-rankit”. An inverse Gaussian transformation is then applied to these B-rankits to get what we call “G-rankits”, that can be used as ordinates for the normal probability plots. Traditional Gaussian-arithmetic sheets are based on these principles.

B-rankits are usually computed with the Equations (1) or (2) or a combination of them, as follows.

$$P_i = \frac{i - \frac{1}{2}}{N} \quad (1) \qquad P_i = \frac{i - \frac{3}{8}}{N + \frac{1}{4}} \quad (2)$$

*where “ P_i ” is the theoretical cumulated probability of the value of rank “ i ”
“ i ” is the rank of value,
and “ N ” is the total number of values.*

Equation (3), proposed by [3] is less commonly used.

$$P_N = \frac{1}{2} \quad P_1 = 1 - P_N \quad P_i = \frac{i-0,3175}{N+0,365} \text{ for other values of } i \quad (3)$$

Table 1 shows an example for $N = 5$:

Table 1. P_i values obtained with Equation (1), Equation (2) and Equation (3) for $N = 5$.

i	1	2	3	4	5
P_i with Equation (1)	0,1	0,3	0,5	0,7	0,9
P_i with Equation (2)	0,1190	0,3095	0,5	0,6905	0,8810
P_i with Equation (3)	0,1272	0,3136	0,5	0,6864	0,8728

These P_i values are then used to compute G-rankits to be used to plot the normal probability plots, using Equation (4).

$$Z_i = \phi^{-1}(P_i) \quad (4)$$

where " P_i " is the theoretical cumulated probability of the value of rank " i "
" i " is the rank of value,
and " Z_i " is the G-rankit to be use as ordinate in normal probability plots.

Table 2 shows the Z_i (G-rankits) obtained with P_i of Table 1.

Table 2. Z_i (G-Rankits) obtained with Equation (4) for P_i values of Table 1.

i	1	2	3	4	5
Z_i from P_i with Equation (1)	-1,282	-0,524	0	0,524	1,282
Z_i from P_i with Equation (2)	-1,180	-0,497	0	0,497	1,180
Z_i from P_i with Equation (3)	-1,140	-0,486	0	0,486	1,140

Equations (2) and (3) are usually regarded as producing better approximations than Equation (1), especially when $N < 10$.

However, these P_i are approximations. Typically, Equation (1) is based on a model where the lower data is uniformly distributed on the interval $]0;1/N]$, the second lower data on the interval $]1/N;2/N]$, etc. ... what is wrong: the lower data can obviously be more than $1/N$.

To compute the exact values, it is necessary to deal with actual distributions of p_i and determine an appropriate average P_i value that causes no bias in the plotting of the normal probability plots.

So, in this study, the following issues are dealt with:

1. How laws of distribution of p_i can be determined;
2. How these distribution laws can be used to compute average P_i values and limits of interval of confidence for them;
3. Whether the approximations for P_i of equations (1), (2) and similar are appropriate;
4. How rankits can be computed, particularly in the cases of the Gaussian distribution law and of the distribution law of standard deviations;
5. How these rankits can be used to compute robust estimations of mean values and standard deviations.

2 Technical backgrounds

2.1 Symbols

The symbols used in this document are listed in Table 3.

Table 3. List of symbols used in this document.

Symbol	Designation and comments
a	Constant usually chosen inside the interval $[0;1]$ to be used in Equation (5)
a_1 and a_2	Constants function of N_r to be used in Equation (33)
A	Slope of the straight line approximating the P_i values
$C_{i,c}$	Centile of the cumulative distribution function for the i^{th} value and the proportion "c"
D_i	i^{th} value of a series of values, ordered in an increasing order
i	Rank of a data or value of among a series of N data or values
IC	Interval of confidence with enlargement coefficient k taken equal to 2
m	Estimate of a mean value
m_i	Estimate of the i^{th} mean value of a series
Med	True median value
Med_i	True median of the i^{th} value of a series
N	Total number of random values
N_r	Number of normally distributed random values in one of the N_s series, used to compute one estimate of a standard deviation
N_s	Total number of estimates of standard deviations computed from N_s series of N_r normal distributed random values
p_i	i^{th} value of an ordered series of random values taken from a uniform distribution on the interval $]0;1[$
P_i	B-rankit, defined as a central value of the distribution of p_i , that can be used in probability plots
s	Estimate of a standard deviation
s_i	i^{th} estimate in an ordered series of a standard deviation estimates
u	Standard uncertainty
U	Enlarged uncertainty, with enlargement coefficient "2"
z_i	i^{th} value of an ordered series of random variables of normal distribution
Z_i	G-rankit, defined as a central value of the distribution of z_i , that can be used in a normal probability plot
z_{ri}	i^{th} value of an ordered series of random variables of a standard deviation probability distribution
ZR_i	S-rankit, defined as a central value of the distribution of z_{ri} , that can be used in a standard deviation probability plot
μ	True value of a mean value
σ	True value of a standard deviation
χ^2_{n-1}	Value of the χ^2 distribution law with $n-1$ degrees of freedom

2.2 Computation of B-rankits

Equations (1) and (2) are in fact examples of the general Equation (5) as follows:

$$P_i = \frac{i - a}{N + 1 - 2a} \quad (5)$$

where " P_i " is the theoretical cumulated probability of the value of rank " i "
" i " is the rank of value,
and " a " is a constant usually pertaining to the interval $[0;1]$.

In the case of Equation (1), $a = 0,5$ and in the case of Equation (2), $a = 3/8$. We can find several proposals of such equations in the literature as, for examples:

- ✚ $a = 0$, corresponding to $P_i = i/(N + 1)$, see § 3.3;
- ✚ $a = 0,3$, corresponding to $P_i = (i - 0,3)/(N + 0,4)$, proposed by [4];
- ✚ $a = 0,3175$, corresponding to $P_i = (i - 0,3175)/(N + 0,365)$, proposed by [3];
- ✚ $a = 0,326$, corresponding to $P_i = (i - 0,326)/(N + 0,348)$, proposed by [5];
- ✚ $a = 1/3$, corresponding to $P_i = (i - 1/3)/(N + 1/3)$, used in BMDP statistical package;
- ✚ $a = 0,4$, corresponding to $P_i = (i - 0,4)/(N + 0,2)$, proposed by [6];
- ✚ $a = 0,44$, corresponding to $P_i = (i - 0,44)/(N + 0,12)$, proposed by [7];
- ✚ $a = 0,5$, corresponding to $P_i = (i - 0,5)/N$, see Equation (1);
- ✚ $a = 0,567$, corresponding to $P_i = (i - 0,44)/(N - 0,134)$, proposed by [8];
- ✚ $a = 1$, corresponding to $P_i = (i - 1)/(N - 1)$, proposed by [3].

In spite of appearance, these equations are equations of straight lines, as it can be seen by decompositions of them as follows: $P_i = \frac{i-a}{N+1-2a} = \frac{1}{N+1-2a} \cdot i - \frac{a}{N+1-2a}$, that is to say in the form of $P_i = A \cdot i + B$. Consequently, when the slope A is known, a can be computed by the Equation (6), as follows.

$$a = 0,5 \cdot (N + 1 - \frac{1}{A}) \quad (6)$$

where " a " is the parameter of Equation (5)
" N " is the number of values,
and " A " is the slope of the straight line approximating the P_i values.

The second important feature of these equations is that $P_{\frac{N+1}{2}} = 0,5$, whatever a and N , as demonstrated here after:

$$P_{\frac{N+1}{2}} = \frac{\frac{N+1}{2} - a}{N + 1 - 2a} = \frac{(N + 1) - 2a}{2(N + 1 - 2a)} = \frac{1}{2}$$

This ensures the expected symmetry of P_i around the central value for i , that is to say $i = \frac{N+1}{2}$.

We can see then that, even if the value of " a " is usually included in the interval $[0;1]$, this is not needed to ensure the consistency of Equation (5).

2.3 Use of normal probability plots for checking the normality of a distribution

Normal probability plots were historically developed (by P.J.P. Henri in the 1880's, see [9] and [10]) for checking the normality of distributions.

It consists in plotting samples quantities in function of theoretical quantities, as shown in Figure 1. When dots are randomly distributed around the straight line, the population can be regarded as normally distributed (or, more accurately worded, “the population cannot be regarded as not normally distributed”).

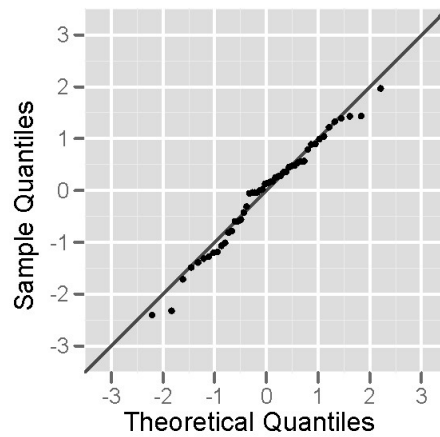


Figure 1: Example of normal probability plot (origin: Wikipedia [10])

Figure 2 shows typical significant deviations from the straight lines.

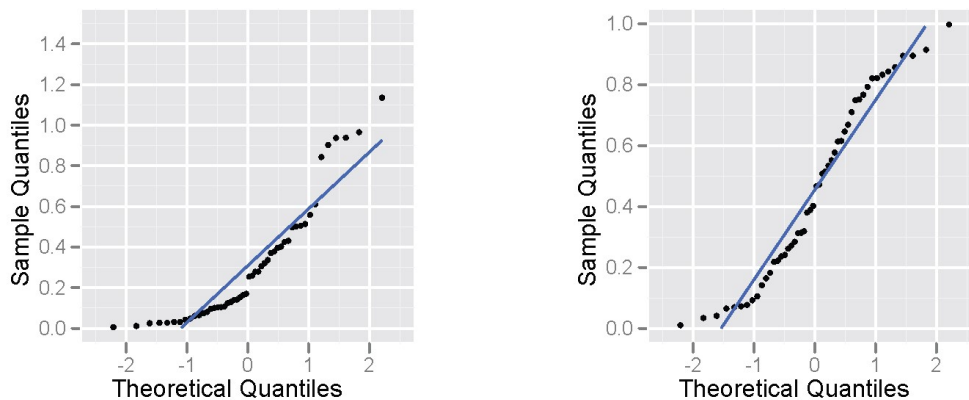


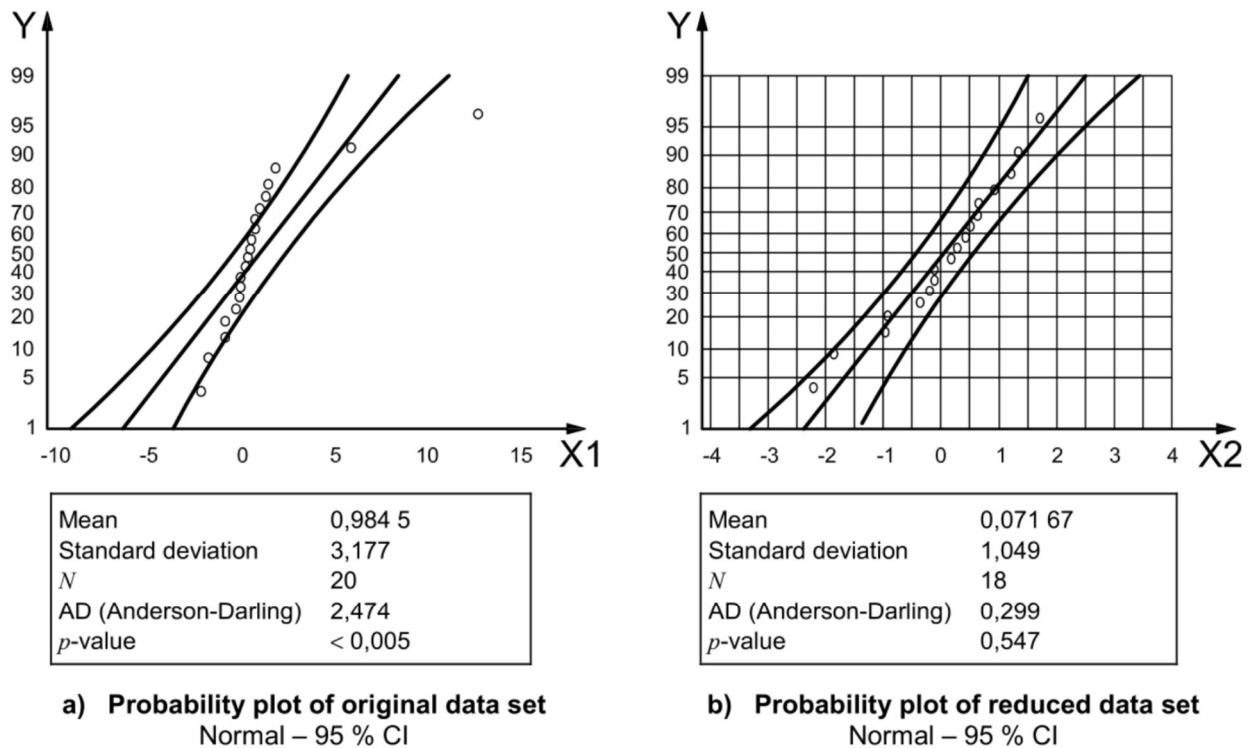
Figure 2: Examples of skewed distribution (left) and of a distribution with kurtosis different from 0 (right) (origin: Wikipedia [10])

More information concerning skewness is available in [11] and concerning kurtosis in [12].

For this use of the normal probability plots, attention is often drawn on the central plots, which are usually quite better determined than the extremum ones. For this reason, the quality of the determination of G-rankits that are used to plot the sample quantiles is of fewer importance than for some other applications.

2.4 Use of normal probability plots for checking the outlying values

When outlying values occur, they obviously appear at the tails of the plot. When a dot is unlikely distant to the straight line, it is likely to be an outlying value. Figure 3 shows an example of outlying values that appear in a normal probability plot.



Key

X1 original data set
X2 reduced data set
Y percent

Figure 3: Example of normal probability plot for detecting outliers (extract from ISO 16269-4 [2])

In this example, the suppression of the 2 largest values changes the distribution from significantly not normal to reasonably fitting a normally distributed one.

By the way, we can see in this example how 10% of outliers can heavily disturb the estimation of mean values and standard deviations.

For this use of normal probability plots, attention is often drawn on the external plots. Theoretically, care should then be taken concerning the G-rankits used to plot the curve. However, when outliers occur, they are usually “dramatically outlying”, so that some approximation on G-rankits (on Y axis in Figure 4) is of low effect compared to the outlying effects (on X1 axis in Figure 4). For this reason, the quality of the determination of G-rankits that are used to plot the sample quantiles is of medium importance compared to other applications.

2.5 Use of normal probability plots for estimating mean values and standard deviations

As shown in Figure 4, in normal probability plots, the theoretical straight line cuts the theoretical $P_i = 0,5$ exactly at the abscissa of the theoretical mean value. In the same way, the distance between the abscissa of theoretical $P_i = 0,841$ and the abscissa of the theoretical $P_i = 0,5$ is the exact theoretical standard deviation of the distribution. In other words, the slope of the theoretical straight line is proportional to the exact theoretical standard deviation of the distribution.

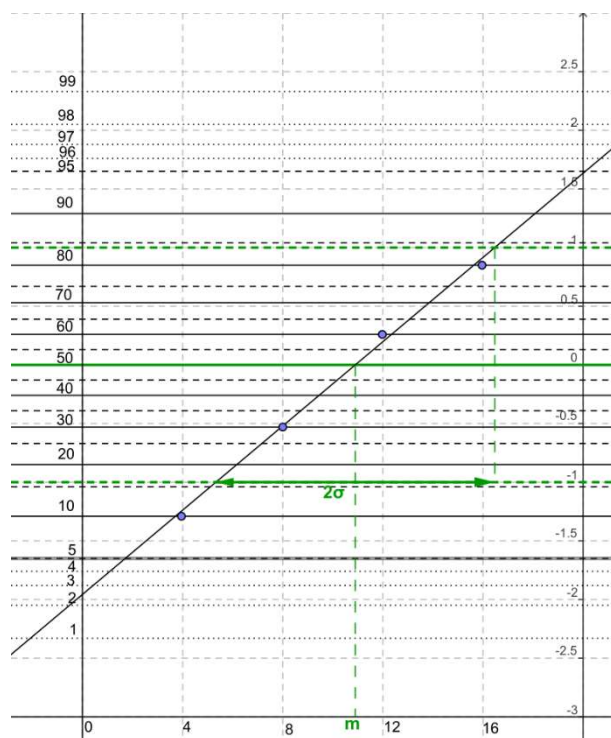


Figure 4: Example of normal probability plot for computing the mean value and standard deviation of a Gaussian distribution (origin: Wikipedia [9])

This Figure 4 seems to be idealised (dots are too well aligned to be realistic). Figure 5 displays 20 random normal probability plots from a same Gaussian distribution ($\mu=10$ and $\sigma=3$): none of them is aligned as displayed in Figure 4.

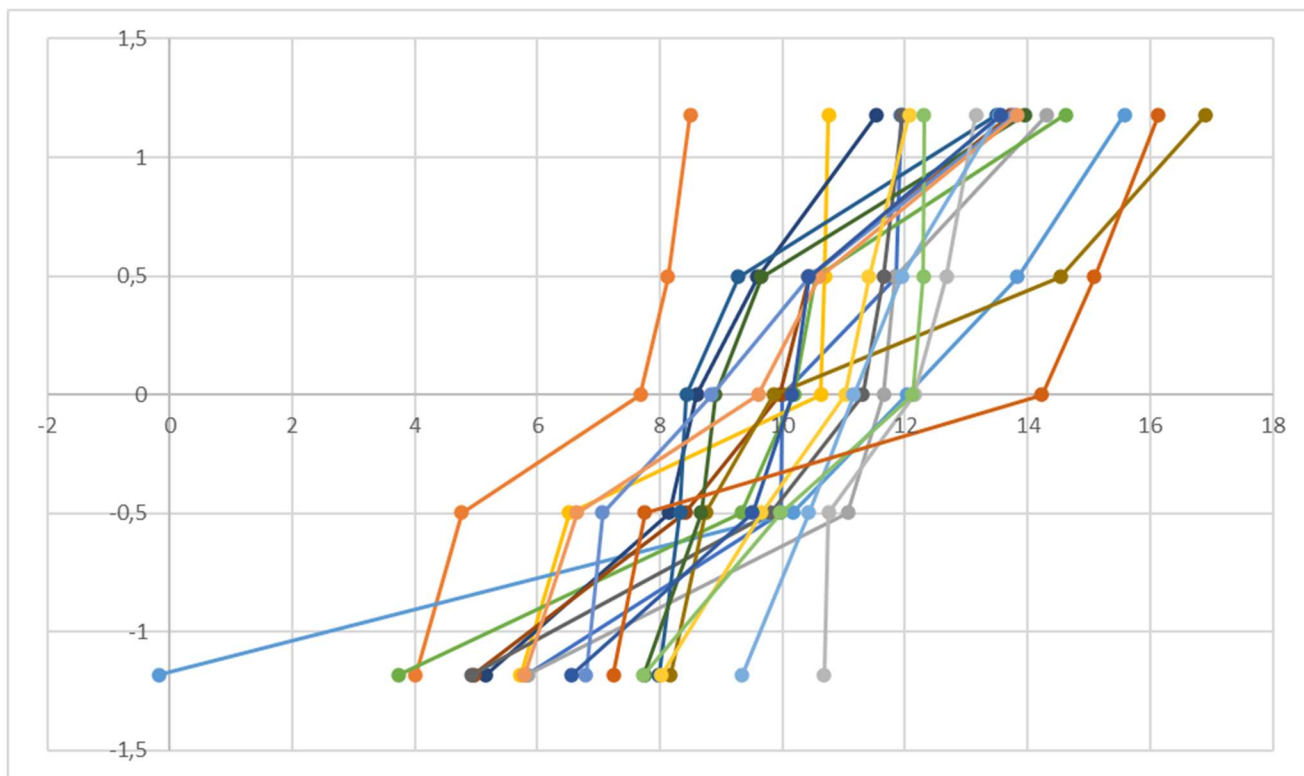


Figure 5: 20 random normal probability plots from the same Gaussian distribution ($\mu=10$ and $\sigma=3$, abscissas: values, ordinates: z_i)

In Figure 4, we can estimate the mean value as being about 11. In the same way, the abscissa for $P_i = 0,841$ is about 16,7, the abscissa for $P_i = 0,159$ is about 5,2, the standard deviation can be estimated as $(16,7-5,2)/2 = 5,7$. Of course, this method is valid only if the distribution can reasonably be regarded as normally distributed.

The quality of the estimation obviously depends on the quality of the drawing of the straight line. Usually, linear regressions are used to draw the straight line, so that μ and σ can be accurately estimated. In the same way, the quality of the regression (that can be estimated visually or by the regression coefficient) and the tools shown in § 2.4 enable to detect and discard outliers and, consequently, eliminate their deleterious effects of the estimations.

For this use of the normal probability plots, external plots are of main importance on the calculations. Care should then be taken concerning the G-rankits used to plot the curve, especially when the number of available values is low (as in Figure 4). When the number of available values is high, it is always possible to use only the central dots to plot the straight line, avoiding then the deleterious effects of the low plotting accuracy of the external dots. When keeping only the one (if n is odd) or two (if n is even) central values, the obtained result is the median and no indication of the scatter is available.

As a conclusion, the number of values taken into account can be adjusted, visually or with the help of an algorithm, to optimise the balance between convergence (using a max number of values) and robustness (using a low number of reliable values).

2.6 Estimation of a standard deviation from repeated series

Ordinates of Figure 4 clearly show how the linear scale (ordinates on the right side) are transformed into cumulated probabilities (ordinates on the left side) via a change of variable using the inverse Gaussian law.

It is obviously possible to use other change of variables using other inverse distribution laws, draw an adequate regression straight line and, in the same way than for the normal probability plot, determine a central trend parameter of the distribution. For an example of that, see [7].

In particular, a standard deviation can be determined from a series of estimates of it. To do so, the first step is to determine the change of variable to operate by using the distribution law of estimates of variances, as follows in Equation (7) (origin: ISO 2854 [13]).

$$(n-1) \frac{s^2}{\sigma^2} \approx \chi_{n-1}^2 \quad (7)$$

where s is the estimate of a standard deviation,

σ is the standard deviation to be estimated,

and n is the number of values used for computing the standard deviation.

The appropriate change of variable can then be deduced by algebraic transformation of Equation (7) as stated in Equation (8).

$$s_i = \sigma \sqrt{\frac{\chi_{n-1}^2(P_i)}{n-1}} \quad (8)$$

where s_i is the estimate of a standard deviation computed from the i^{th} series,

σ is the standard deviation to be estimated,

P_i is the theoretical cumulated probability of the value of rank " i "

and n is the number of values used for computing the standard deviation.

Figure 6 shows an example of linear regression of 20 series of 5 values with mean value equal to 10 and standard deviation equal to 3 (see Figure 5). To build up this figure, S-rankits were determined as theoretical s_i/σ ratios (see Equation (8)), expected in function of the rank “i” of the estimates s_i of the true standard deviation σ . The 20 corresponding estimates of the standard deviation scatter from 1,12 to 6,18, which is consistent with Equation (8). The straight of linear regression cuts the ordinate “1” at 3,0, which is the true standard deviation.

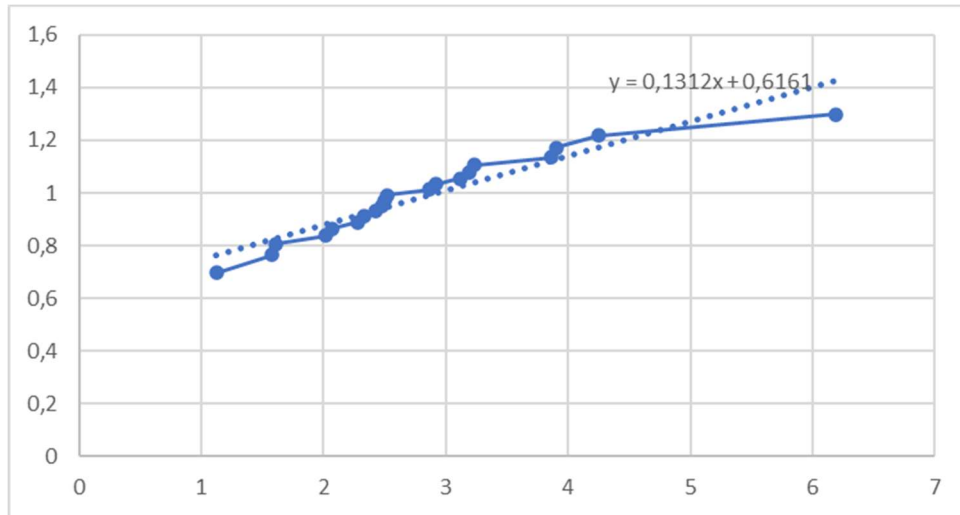


Figure 6: Example of standard deviation probability plot of a series of 20 standard deviation estimates from 5 values each.
Abscissas: s_i , ordered estimates of σ , Ordinates: S-rankits.

As for Gaussian distributions, we can adjust the number of values to take into account, visually or with the help of an algorithm, to optimise the balance between convergence (using a max number of results) and robustness (using a low number of robust values).

This method can be particularly useful when many series of little quantities of values are available, for example when the checking of product conformity is performed batch by batch. The inner batch standard deviation can be accurately estimated, without any effect of the inter-batches scatter.

2.7 The binary search algorithm

The binary search algorithm enables to find out values $y = f(x)$ when the function f^{-1} is difficult or impossible to express in an algebraic way. In practice, it consists in:

1. Define an interval $[min;max]$ which is supposed to contain “y”. $f(x)$ shall be continuously increasing or continuously decreasing on the $[min;max]$ interval;
2. Compute the central value C_1 of the interval using the equation $C_1 = \frac{min+max}{2}$;
3. Compute $f(C_1)$ and compare it to “y”;
4. According to the result of step 3 and whether $f(x)$ is increasing or decreasing on $[min;max]$, decide whether “x” for which $f(x) = y$ is greater or lower than C_1 ;
5. Select the interval $[min; \frac{min+max}{2}]$ or $[\frac{min+max}{2}; max]$ in which “y” is supposed to be located and go to step 2, in order to compute C_2 ;
6. After N steps, an approximation to the nearest $\frac{1}{2^N}(max - min)$ of “y” is given by the value C_N .

This method can easily be used on cumulative probability distribution law, because they are, by definition, always monotonously increasing on the interval $]0;1[$.

This method was used to find roots of polynomials needed to solve questions raised in § 3. When it can be used, it requests volumes of calculations significantly lower than the Monte-Carlo method.

2.8 The Monte-Carlo method

The Monte-Carlo methods are a large category of algorithms that use random numerical realisations of a given model. They are often used to solve mathematical or physical problems, difficult or impossible to solve by other methods. For a survey of the history and applications of the Monte-Carlo methods, see for example [14].

Hard calculations are needed to solve several of the issues of this document. In order to simplify these calculations, we used the Monte-Carlo method. In the frame of this study, determining centiles, medians or mean values of distributions request to solve some difficult integrals and to find zeros of polynomial equations. To avoid this, large series of random realisations are created, what enables to compute these centiles, medians or mean values.

However, using Monte-Carlo methods requests to use a model that represents reasonably well the situations that we want to deal with. To achieve this, an appropriate modelling is needed. This is obviously not a problem in the present case. As a matter of fact, in the frame of this study, the same appropriate modelling is also needed to establish the equations to solve.

Using the Monte-Carlo methods also requests to use random input values. When several random values are necessary to produce one Monte-Carlo result and when correlations between them apply in real life, these correlations must be incorporated in the input values of the computations. That can be a bit difficult to do properly. In our case, the Monte-Carlo results are of only one type: Gaussian distributed numbers. Consequently, no correlation is to be feared.

To assure the validity of the conclusions, the random series need to be numerous enough, depending on many factors. In our study, we computed series of 10^5 to 10^7 numbers for each situation. Each of these series was divided in sub-groups. This enables us to compute the repeatability of the parameters that we are determining. This repeatability standard deviation is then used to determine an interval of confidence (IC) for each of the determinations, with an enlargement coefficient equal to 2. We decided to stop the Monte-Carlo processes when we considered that the IC is small enough for each particular issue to solve. These IC are provided in the results when relevant and drove the rounding of the results that are provided by this study.

3 Laws of distribution of p_i

3.1 Introduction

It is reminded that p_i is the probability of the i^{th} value of a series of N numbers uniformly distributed on the interval $]0;1[$. It is defined as follows:

- ✚ Each piece of data D_i is the realisation of a random variable d_i that describes the totality of values that the piece of data D_i can be ;
- ✚ The probability P_i is the cumulated probability D_i . It is a realisation of the random variable p_i (that, by definition of probabilities, is defined on the interval $]0 ;1[$ and comes from the random variable d_i ;
- ✚ When $N = 1$, the distribution of p_1 is uniform on the interval $]0;1[$;
- ✚ When $N > 1$, for a given value of p_i (abscissa in Figure 7), the probability that p_j value is lower than this value is exactly p_i ;

- The repetition of this phenomena makes that the distribution of p_i values is binomial, with a total number of trials equal to $N-1$ and a number of successes equal to i .

The validity of this model was confirmed by the Monte-Carlo method.

Figure 7 shows the distributions of density of p_i for $N = 1$, $N = 2$ and $N = 3$.

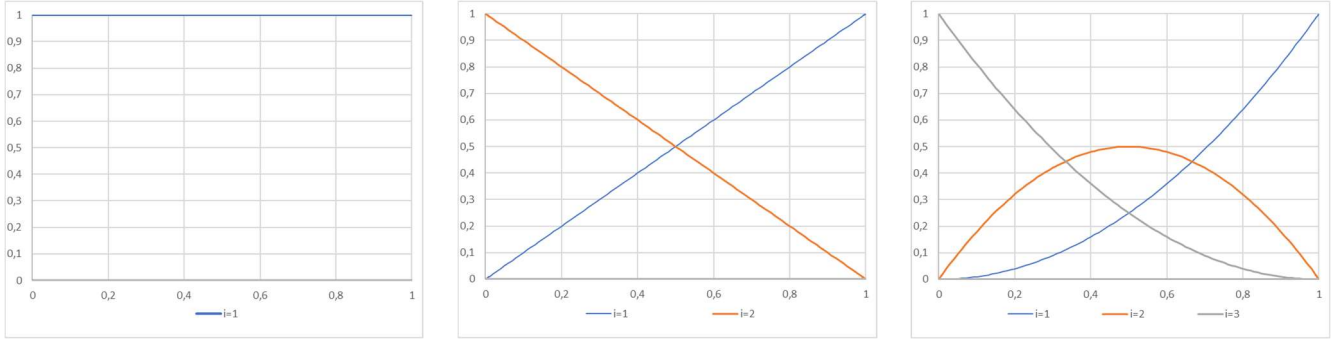


Figure 7. Distributions of density of p_i for $N = 1$, $N = 2$ and $N = 3$

3.2 Equations for p_i , mean value, centile and median values

As curves of Figure 7 show the density of probability of being the i^{th} p value among a series of N ones laying between 0 and 1 in function of this value:

- The sum of all p_i is equal to 1. This is because, in any case, all p_i are single i^{th} values;
- The area under all of these curves is equal to $1/N$, because sum of all p_i is equal to 1. Consequently, the densities of probability shown in Figure 7 need to be multiplied by N in order to normalise the distribution.

Consequently, Equation (9) here after represents the distributions of p_i .

$$d(p_i) = \binom{N}{i-1} \cdot p^{i-1} \cdot (1-p)^{N-i} \quad (9)$$

where “ i ” is the rank of value

“ $d(p_i)$ ” is the density of probability for a given “ p ” to be the i^{th} value “ p_i ” of the series

“ N ” is the total number of values of the series.

For example, the density of probability for a given value to be:

- As shown in Figure 7 (right curve), the 2nd of 3 values pertaining to the interval $]0;1[$ is $3 \cdot p_2 \cdot (1 - p_2)$.
- The 3rd of 4 values pertaining to the interval $]0;1[$ is $12 \cdot p_3^2 \cdot (1 - p_3)$.

The symmetry of the Binomial distribution (in this case, $i-1$ and $N-i$) allows the following statements:

- $d(p_i) = d(1-p_{N-i})$;
- When N is an odd number, the distribution $d(p_i)$ of the central i value ($i = (N + 1)/2$) is symmetrical and centred on $p = 0,5$. Consequently, in those cases, both mean value and median value are equal to 0,5. Figure 7 (right curve) provides an illustration of this for $N = 3$ and $i = 2$.

From this distribution function, mean value and centile values can be computed with Equations (10) and (11). Equation (11) also enables to compute the median, regarded as the 50% centile value.

$$m_i = \int_0^1 p_i \cdot d(p_i) dp \quad (10)$$

$$C_{i,c} = \int_0^c d(p_i) dp \quad (11)$$

where “ $d(p_i)$ ” is the distribution function described by Equation (9)

m_i is the mean value of the distribution of rank “ i ”

$C_{i,c}$ is the centile of the cumulative distribution function to be reached (example, 50% for the median value)

“ c ” is the p_i value for which the desired centile is reached.

3.3 Calculation of the mean values of p_i distributions

Cases of $i = N$ and of $i = 1$

In the case of $i = N$, Equation (10) becomes: $m_N = N \cdot \int_0^1 p^N dp$, then $m_N = N \cdot [\frac{p^{N+1}}{N+1}]_0^1$, and as $m_N = 1 - m_1$ (see above):

$$m_N = \frac{N}{N+1}$$

$$m_1 = \frac{1}{N+1}$$

where m_N is the mean value of the larger value among all the p_i

where m_1 is the mean value of lower value among all the p_i

“ N ” is the number of values.

Other cases ($i \neq 1$ and $i \neq N$)

By using Equation (10), it can be found that $m_i - m_{i-1} = \frac{1}{N+1}$, whatever i . We can then conclude Equation (12):

$$m_i = \frac{i}{N+1} \quad (12)$$

where m_i is the mean value of the i^{th} p value;

“ N ” is the number of p_i values.

This corresponds to Equation (5) with “ a ” = 0, and this is consistent with Filliben 1975 [3].

Conclusion:

Equation (12) enables us to find out easily the mean value of the distribution of p_i , whatever N and i .

3.4 Calculation of the centiles and median values of p_i distributions

Cases where $i = N$ and $i = 1$

When $i = N$, Equation (11) becomes: $C_{N,c} = N \cdot \int_0^c p^{N-1} dp$, then $C_{N,c} = N \cdot [\frac{p^N}{N}]_0^c = c^N$. We can conclude Equation (13) from it, as follows:

$$C_{N,c} = c^N \quad (13)$$

where $C_{N,c}$ is the “ c ” centile value of the N^{th} p value;

“ c ” is the considered centile.

And, in the case of the median, for which $c = 0,5$:

$$Med_N = 0,5^{\frac{1}{N}} \quad (14)$$

In the same way than for the mean value, as $p_i = 1 - p_{N-i}$ (see above)

$$Med_1 = 1 - 0,5^{\frac{1}{N}} \quad (15)$$

These values were proposed by Filliben 1975 [3] as P_i recommended values, see Equation (3).

Using the finite development of polynomials, it can be demonstrated that:

$$Med_1 \rightarrow \frac{\ln(2)}{N} \approx \frac{0,69315}{N} \text{ when } N \rightarrow \infty \quad (16)$$

Other cases ($i \neq 1$ and $i \neq N$)

In those cases, Equation (11) becomes:

$$C_{i,c} = \binom{N}{i-1} \int_0^c p^{i-1} \cdot (1-p)^{N-i} dp \quad (17)$$

To be solved, this equation requests the development of $p^{i-1} \cdot (1-p)^{N-i}$ which includes $N+1$ terms, what becomes quickly fastidious. Fortunately, computer calculation programs (including spreadsheets) include a “cumulative” Binomial function that computes $C_{i,c}$ values as function of p , i and N , without any need to find out the primitive function of Equation (17).

When $i = N-1$, Equation (11) becomes:

$$\begin{aligned} C_{i,c} &= \binom{N}{N-2} \int_0^c p^{N-1} \cdot (1-p) dp, \text{ and then} \\ C_{i,c} &= N \cdot (N-1) \cdot \left[\frac{p^N}{N} - \frac{p^{N-1}}{N-1} \right]_0^c, \text{ and then} \\ (N-1) \cdot c^N - N \cdot c^{N-1} - C_{i,c} &= 0 \end{aligned} \quad (18)$$

where $C_{i,c}$ is the $C\%$ centile of the i^{th} p value;
“ N ” is the number of p_i values.

For each $C_{i,c}$, the polynomial Equation (18) needs to be solved to find its root “ c ” pertaining to the interval $]0;1[$. According to the mathematical “group theory”, this equation cannot be solved directly as soon as $N-1 > 4$. An algorithm to compute approached values is then necessary to find the solutions. We used the binary search algorithm which is well adapted in this situation (see § 2.7).

For example, to compute the median value in the case of $N=5$ and $i=4$, the equation $5 \cdot Med^4 - 4 \cdot Med^5 - 0,5 = 0$ needs to be solved. The approximate solution is $Med = 0,6862$, found with the binary search algorithm.

3.5 Use of p_i distributions in probability plots

Figure 8 shows how a random value distributes to become an i^{th} value, and how this i^{th} value is transformed into a z -position in the case of a Gaussian distribution. For doing it, the case of $N=7$ was taken as an example.

Ordinates of Figure 8a represent the rank position of all the potential positions of values, linearly ranked. Figure 8b shows through orange arrows how the median of all potential values is transformed into a corresponding Gaussian G-Rankit. In order to avoid complexity in the figure, arrows are shown only for the case $i=2$. But the same operations can be performed for all other i , enabling to find out centiles 10%, 50% (median) and 90% for each of these i values. In this example, it can be seen on Figure 8c that:

- ✚ The 10% - 50% - 90% centiles values of p_2 when $N = 7$ are 0,095 – 0,23 – 0,45. In other words, the median of p_2 is 0,23 and 80% of p_2 belong to the interval [0,095;0,45];
- ✚ The 10% - 50% - 90% centiles values of z_2 when $N = 7$ are – 1,41 – -0,74 – -0,13. In other words, the median of z_2 is 0,23 and 80% of z_2 belong to the interval [-1,41;-0,13].

This example clearly shows that:

- ✚ When plotting probability plots, the true p_i of each value is unknown. It is then necessary to use P_i values that are some kind of appropriate “average” value that represents the p_i distribution. In the best cases, a segment representing the IC (Interval of Confidence) on P_i could be plotted instead of a single dot, but this is difficult to achieve in practice because the limits of the IC depend on both i and N and request hard calculations to be computed;
- ✚ Both Binomial transformation and Gaussian have asymmetry effects but opposite to each other. These two asymmetries then more or less compensate each other, so that in the median result of z_2 is not far from being at the centre of the $IC_{80\%}$ interval. This is a chance situation and, as this obviously strongly affect the choice for appropriate P_i among all possibilities, this needs to be carefully examined when such a choice must be done;
- ✚ When an asymmetry effect is present on the final z_2 result, the mean value m_i is not a good candidate for P_i . This is for example particularly the case of the low values of the S-distributions (see § 2.6), for which asymmetries of step 1 curve and step 2 curve cumulate instead of compensate.

Figure 8a. Cumulative distributions of p_i for $N = 7$
(abscissas are p_i , green curve: $i=2$)

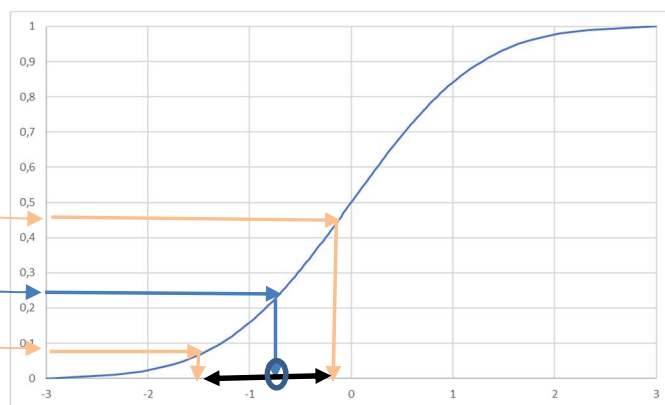
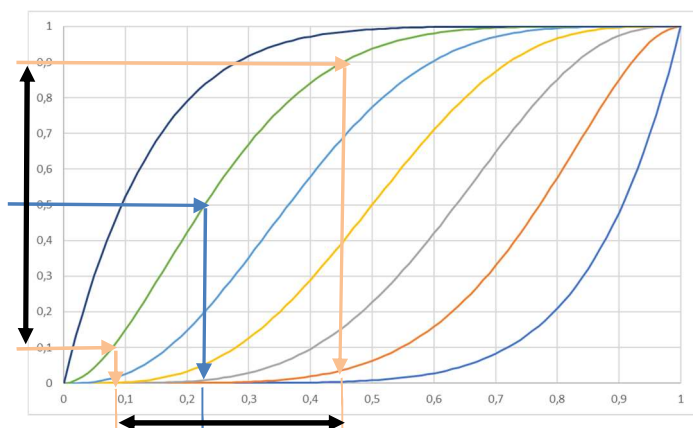


Figure 8b. Gaussian cumulative distribution

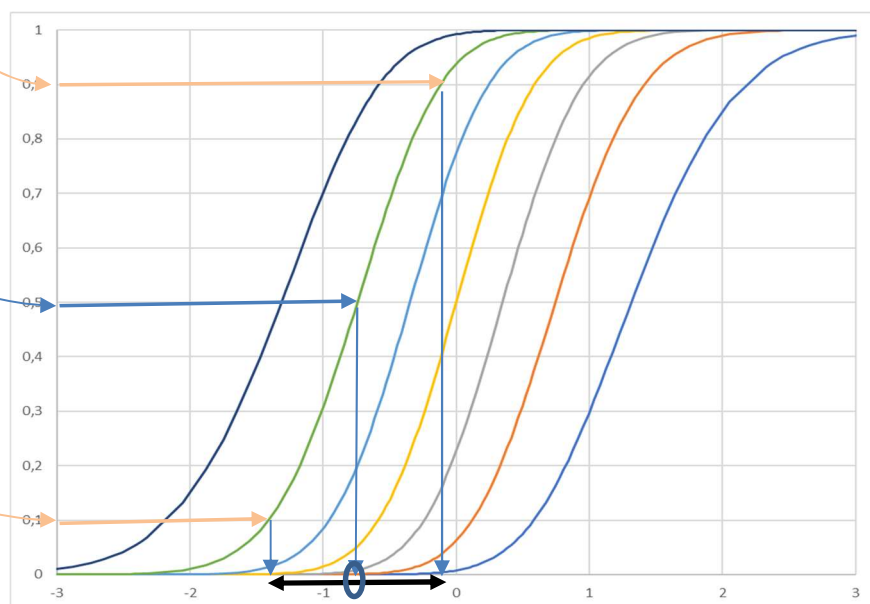


Figure 8c: Distributions of z_i for $N = 7$, $i = 1$ to 7 for a Gaussian distribution

3.6 Conclusions

Probability distributions of ordered random values from uniform distributions can be modeled by Binomial distributions (see Equation (9)).

Mean values of P_i can be determined with Equation (5) with “a” = 0.

Approached values of the centile values, and among them the median values of the p_i distributions can easily be determined with the binary search algorithm.

When the type of distribution of the population is unknown, median values are better adapted to determine appropriate P_i than mean values. Centile values of p_i distributions (typically $c = 0,1$ and $c = 0,9$) can be used to determine intervals of confidence for P_i , whatever the form of the distribution.

4 Design of experiments

4.1 Introduction about appropriate P_i to use for probability plotting

It can be concluded from the upper that:

- ✚ The choice of a P_i (B-rankit the i^{th} value) representing the p_i distributions is needed for probability plotting;
- ✚ The mean values of p_i distributions are easy to compute but are probably not the most appropriate choices for P_i ;
- ✚ The median values of p_i distributions are in most cases probably more appropriate but are more difficult to determine;
- ✚ Equations generally used (see § 1) are linear approximations and could perhaps be improved.

4.2 Design of experiments for this study

With respect to the goals of this study, the following enquiries were performed:

- ✚ Determine the p_i distributions for $N = 2$ to 30. These distributions are characterised by their mean values, and centiles 0,5%, 1%, 5%, 10%, 50% (median), 90%, 95%, 99% and 99,5%. This was performed by using Equation (11) for mean values and binary search algorithm for centiles;
- ✚ Determine the z_i distributions for $N = 2$ to 30 for the Gaussian distribution, characterised by their mean values, and centiles 5%, 50% (median) and 95%. This was performed by using Monte-Carlo method;
- ✚ Determine the z_{r_i} distributions for $N = 2$ to 30 for the standard deviation distribution, characterised by their quadratic mean values, mean values, and centiles 5%, 50% (median), and 95%. This was performed by using Monte-Carlo method;
- ✚ Check for each case which of the present usual methods are the best appropriate to determine P_i (B-rankit);
- ✚ If necessary, propose adequate equations to determine better approximations of P_i and their intervals of confidence.

5 Results and discussions

5.1 Determination the p_i distributions for $N = 2$ to 30

5.1.1 Results of the p_i distributions

Detailed results of the p_i distributions for $N = 2$ to 30 are presented in annex, Table A1. Figure 9a to Figure 9d show the mean value, the median value and different centiles of p_i for different values of N .

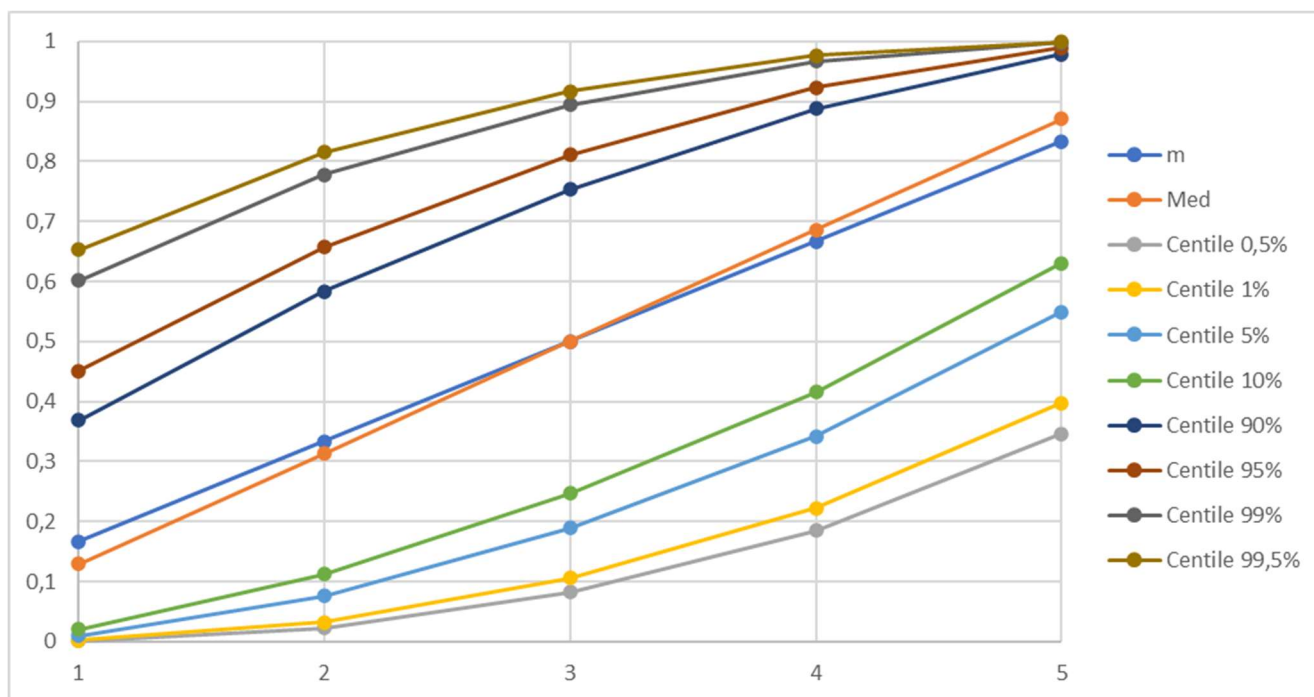


Figure 9a. Mean value "m", median value "Med" and different centiles of p_i as function of i for $N=5$

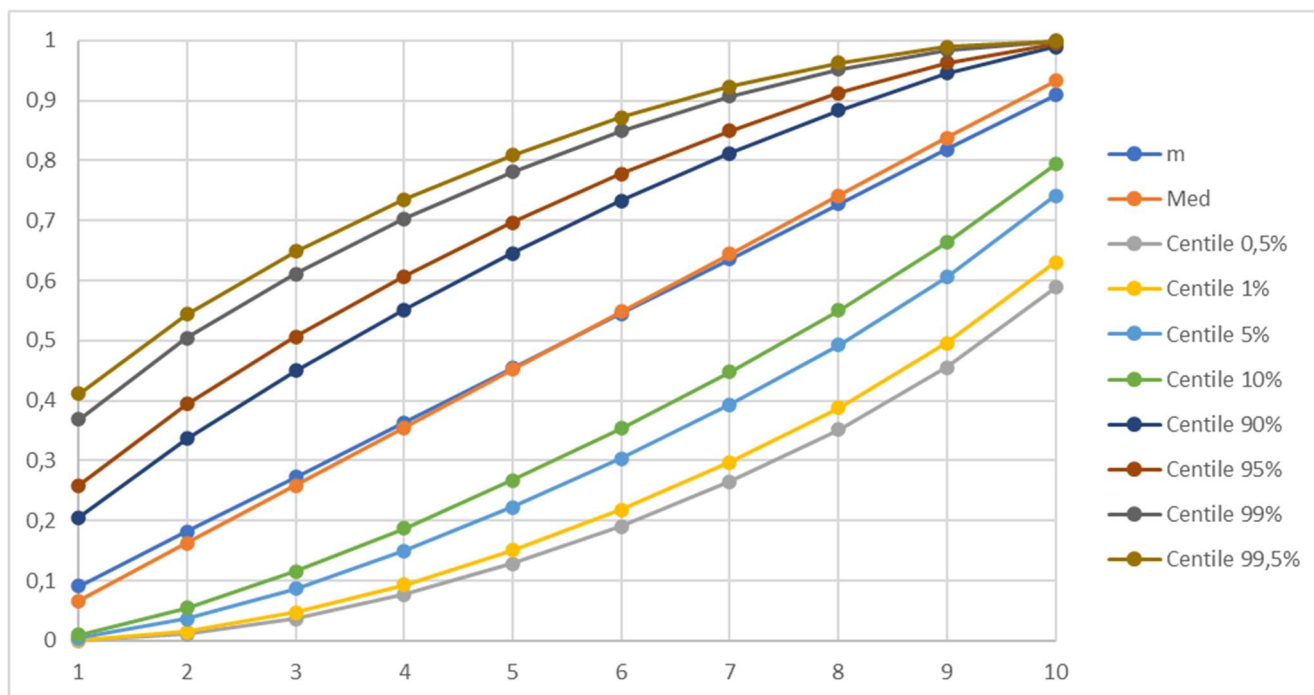


Figure 9b. Mean value "m", median value "Med" and different centiles of p_i as function of i for $N=10$

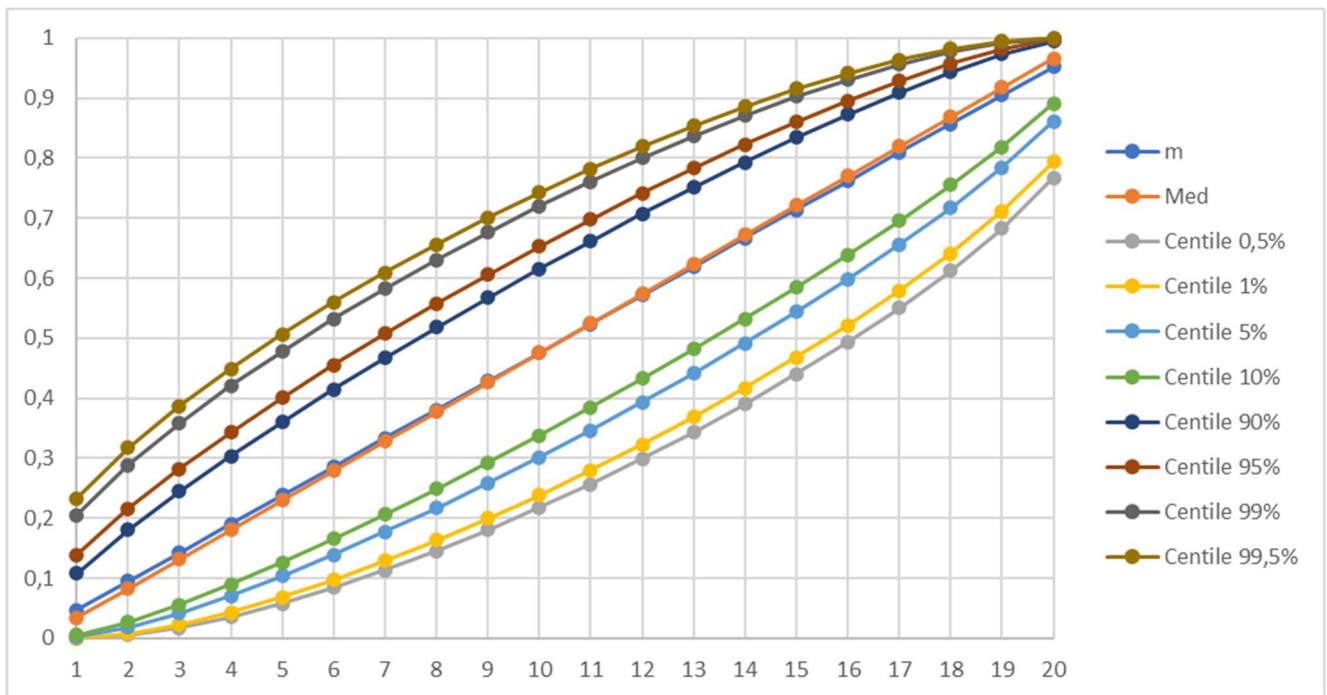


Figure 9c. Mean value "m", median value "Med" and different centiles of p_i as function of i for $N=20$

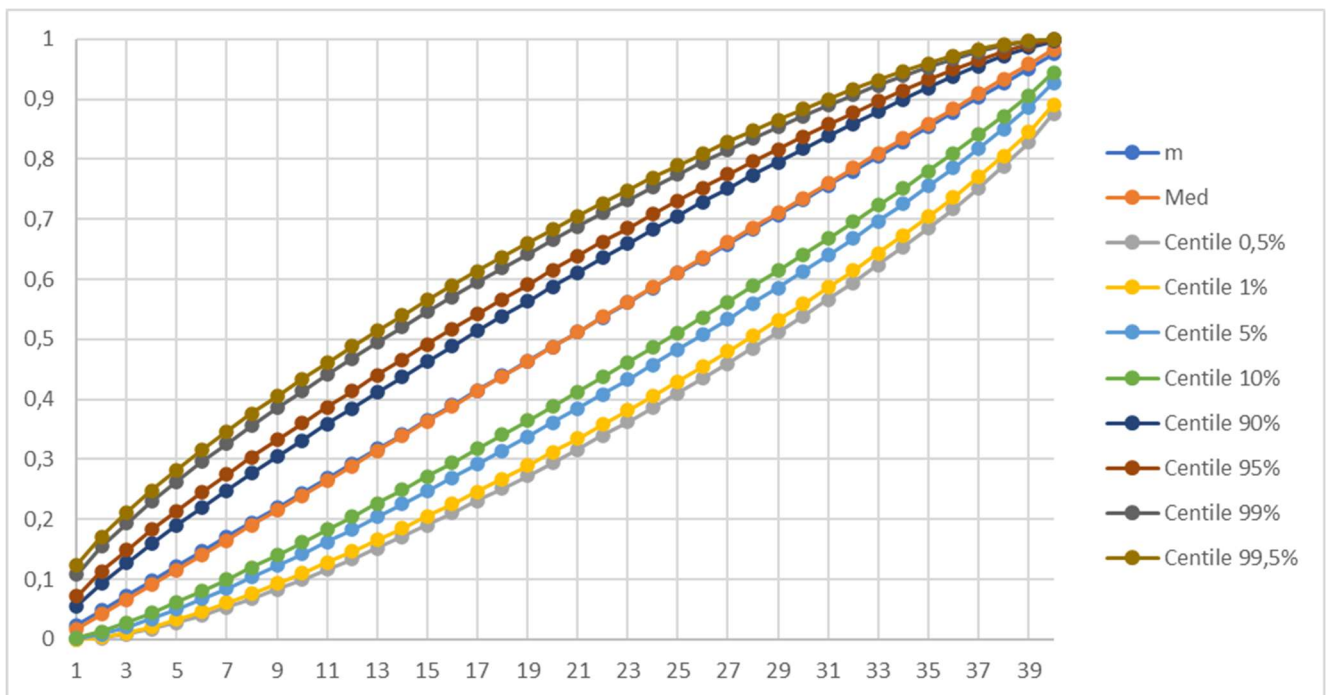


Figure 9d. Mean value "m", median value "Med" and different centiles of p_i as function of i for $N=40$

These figures clearly show that:

- ✚ Both mean values and median values are equal to 0,5 for $i = \frac{N+1}{2}$, whatever N , when i is an odd number. When i is an even number, both curves of median values and median values cut the ordinate 0,5 for the abscissa $\frac{N+1}{2}$;
- ✚ Both mean values and median curves are very close to straight lines. The slopes of median lines are slightly greater than the slopes of mean lines. They tend to same values when N is increasing;

- Due to the symmetry of the Binomial distribution law, $C_{\alpha,i} = C_{1-\alpha,N-i}$, whatever the percentage α of the centile, i and N .

5.1.2 Linear approximations of the median values in function of i and N

Equations using statements of § 3.2

Med_1 (median value for $i=1$) and Med_N (median value for $i=N$) can be easily found by using equations (14) and (15), whatever N .

As seen in § 3.4, the computations of Med_i values are quite more complicated for i values varying from $i=2$ to $i=N-1$. However, these median values appear to distribute very closely to the straight line connecting Med_1 and Med_N , as shown in Figure 10.

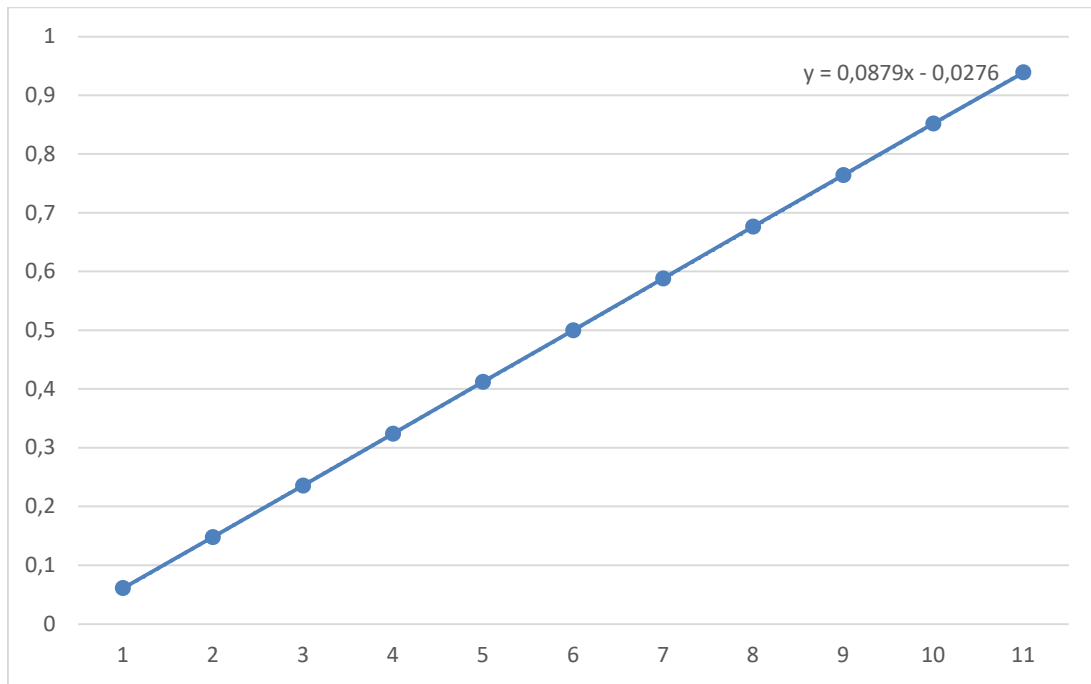


Figure 10: Median value of p_i as function of i for $N=11$

Consequently, Med_i can be computed using the following equation $Med_i = \frac{Med_N - Med_1}{N-1} \cdot (i-1) + Med_1$ where $Med_N = 0,5^{\frac{1}{N}}$ and $Med_1 = 1 - 0,5^{\frac{1}{N}}$, see equations (14) and (15). The general equation (19) can be found by combining them as follows:

$$Med_i = \frac{2 \times 0,5^{\frac{1}{N}} - 1}{N-1} \cdot (i-1) + (1 - 0,5^{\frac{1}{N}}) \quad (19)$$

By combining with Equation (6), Equation (19) can also be expressed in the form of Equation (5) (i.e. $P_i = \frac{i-a}{N+1-2a}$) with the “ a ” coefficient determined with the Equation (20), as follows.

$$a = 0,5 \cdot \left(N + 1 - \frac{N-1}{2 \times 0,5^{\frac{1}{N}} - 1} \right) \quad (20)$$

This equation (19) produces values of Med_i values to the nearest 0,001 for at least N up to 100.

When $N \geq 23$, Equation (16) can be used to simplify Equation(20), keeping the accuracy of determination of Med_i (nearest 0,001):

$$a = 1 - \ln(2) \left(1 + \frac{1}{N} \right) \quad (21)$$

It is reminded that $\ln(2) \approx 0,69315$.

When N is large, it can easily be found from Equation (21) that “a” tends to 0,3069.

When a better approximation is needed, the slope of the straight line can be corrected with an empirical coefficient that we determined to be equal to $K_N = 1 + \frac{1}{20N+100}$. Equation (20) needs then to be corrected into Equation (22), as follows:

$$a = 0,5 \cdot \left(N + 1 - \frac{N - 1}{\left(2 \times 0,5^{\frac{1}{N}} - 1 \right) \left(1 + \frac{1}{20N + 100} \right)} \right) \quad (22)$$

Equation (5) with “a” values from Equation (22) produces values of Med_i values to the nearest 0,0002 for at least N up to 100. This accuracy is as good as if a polynomial equation of degree 3 were used to approximate P_i as function of N instead of a straight line as proposed here upper.

Figure 11 shows the evolution of “a” as a function of N :

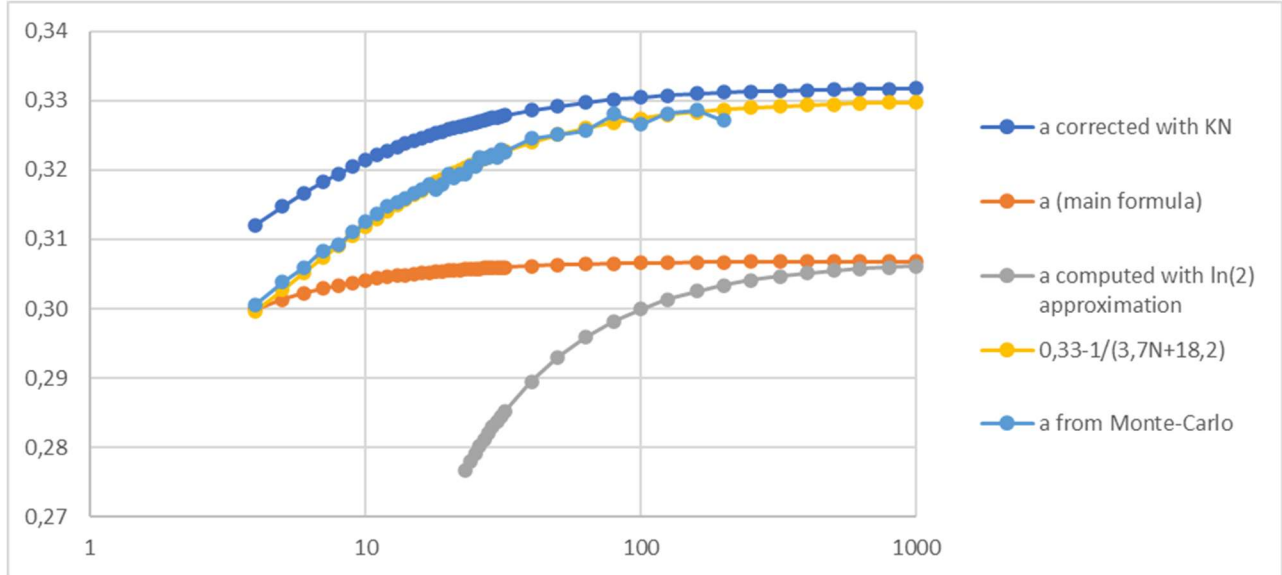


Figure 11: Values of “a” as a function of N , with 5 different calculation methods.

Conclusion: When N is large, “a” tends to 0,3069 with Equation (20) and to 0,331 with Equation (22).

Determination of “a” by linear regression, with the Monte-Carlo method

The determination “a” was also performed using regression parameters of P_i defined as median values of p_i distributions. To do so, series of p_i values were randomly created and ordered. Regression lines of corresponding P_i were determined. The “a” values for these regression lines were then determined. This enables to determine overall mean values of “a”, as a function of N . Table 4 and Figure 12 provide the results of these determinations.

Table 4. Mean values of “a” of Equation (5) obtained by regression for the p_i distribution, as a function of N.

N	"a"	2u	N	"a"	2u	N	"a"	2u
4	0,30062	0,00052	18	0,31709	0,00071	32	0,32262	0,00094
5	0,30382	0,00051	19	0,31789	0,00075	40	0,32455	0,00148
6	0,30594	0,00053	20	0,31931	0,00078	50	0,32504	0,00185
7	0,30837	0,00055	21	0,31880	0,00081	63	0,32560	0,00102
8	0,30932	0,00060	22	0,31937	0,00087	80	0,32808	0,00145
9	0,31101	0,00057	23	0,31945	0,00086	100	0,32658	0,00127
10	0,31254	0,00055	24	0,32045	0,00082	125	0,32814	0,00222
11	0,31370	0,00053	25	0,32043	0,00075	160	0,32864	0,00364
12	0,31470	0,00058	26	0,32186	0,00080	200	0,32712	0,00356
13	0,31525	0,00061	27	0,32167	0,00060	250	0,31947	0,00629
14	0,31583	0,00065	28	0,32178	0,00085	320	0,32538	0,00605
15	0,31654	0,00070	29	0,32210	0,00088	400	0,32158	0,00707
16	0,31718	0,00063	30	0,32184	0,00115	500	0,32289	0,00883
17	0,31798	0,00068	31	0,32289	0,00092			

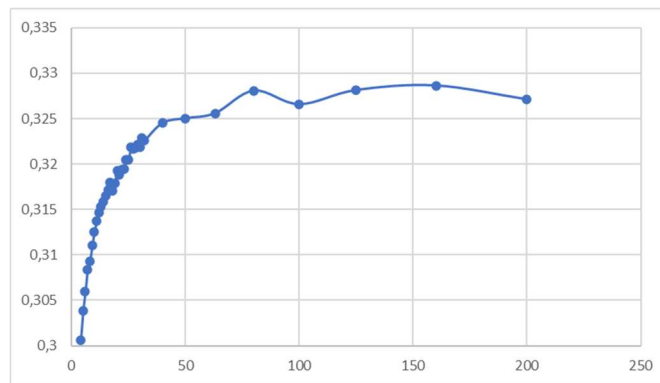


Figure 12: Mean values of “a” of Equation (5) obtained by regression for the p_i distribution, as a function of N.

For $N > 200$, median values of “a” tend to a limit of about 0,33. Amounts of calculation for getting accurate determinations become huge, and there is no need for them because, in these situations, a variation of 0,01 in “a” is of almost no effect in the further use of P_i values. The slight irregularities that can be observed for large values of N are linked to this lack of accuracy.

Equation (23) provides accurate approximations of results of Table 4. Corresponding “a” values appear of enough accuracy compared to $N + 1 - 2a$ (denominator of Equation (5)), whatever N.

$$a = 0,33 - \frac{1}{3,7 \cdot N + 18,2} \quad (23)$$

Where: “N” is the total number of values of the series.

Figure 11 shows that Equations (22) and (23) lead to very close results when $N \rightarrow \infty$.

In addition to median values, this method provides standard deviations of “a”. This is interesting because it gives an information about the scatter with which “a” can vary around its median value. Figure 13 displays the results of it.

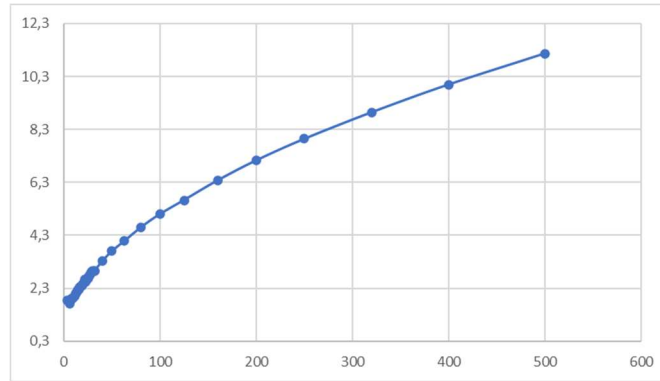


Figure 13: Standard deviations of “a” of Equation (5) obtained by linear regression for the p_i distribution, as a function of N.

Equation (24) appeared to provide a correct approximation of standard deviations of “a” values as a function of N, for $N > 8$.

$$\sigma_a = \sqrt{0,25 \cdot N + 1,3} \quad (24)$$

Where: “N” is the total number of values of the series.

In all cases, $\sigma_a > 1,7$, which means that the width of IC (with $k=2$) on “a” values is always more than $\pm 3,4$, what is huge compared to differences between recommended values for it ($0 \leq i \leq 1$, see § 2.2).

5.1.3 Polynomial approximations of the centile values in function of i and N

Determination of centile values:

As it can be clearly seen on Figure 9, centiles values cannot be accurately approximated by straight lines, contrarily to median values. To overcome this difficulty, we checked approximations with polynomials of degrees varying from 2 to 6, as follows:

$$C_{i,N,\alpha} = c_2 \cdot i^2 + c_1 \cdot i + c_0$$

$$C_{i,N,\alpha} = c_6 \cdot i^6 + c_5 \cdot i^5 + c_4 \cdot i^4 + c_3 \cdot i^3 + c_2 \cdot i^2 + c_1 \cdot i + c_0$$

where:

i is the rank of the value in an ordered series of N terms

c_j are coefficients depending on N and α

α is the cumulative proportion to be reached.

This attempt produced results that cannot be easily modeled. The best way to determine them is to compute them with the binary search algorithm, as explained in § 2.7.

Determination of the 5% α -centile values:

An attempt to compute a formula providing a_i coefficients as function of N was also made for the α -centile 5%, for values from $N=3$ to 100.

Equation (25) is the result of these calculations:

$$C_{i,N,5\%} = c_2 \cdot i^2 + c_1 \cdot i + c_0 \quad (25)$$

where:

$$c_2 = (0,6728 \times N + 1,0942)^{-0,395}$$

$$c_1 = \frac{(0,31966 \times N - 0,1196) \times (N - 5,321)}{N^{2,8}}$$

$$c_0 = \frac{(0,07291 \times N - 0,3387) \times (N - 3,986)^{-0,2667}}{N}$$

These equations provide approximated values of $C_{i,N,5\%}$ to the nearest 0,01 when $i < 0,9.N$. When $i > 0,9.N$ approximations of $C_{i,N,5\%}$ are approached to the nearest 0,02.

Note that in most cases, for $i < 0,1.N$ approximations to the nearest 0,01 is not at all enough to provide useful information. Typically, if these centiles are used for an inverse Gaussian law, low values of i are used in the tails of the Gaussian distribution where each 0,001 increment has a huge impact on the calculated interval of confidence. In those cases, the binary search algorithm should always be used.

Intervals of confidence on p_i for median values of i :

Equation (25) with $i = N/2$ can obviously be used to compute the intervals of confidence (90% bilateral) on p_i to the nearest 0,01. For other $1-\alpha$ values or for better precision, results of tables A1 and A2 in annex can be used. Care shall be taken that these tables provide centiles (unilateral), while intervals of confidence are bilaterally bounded. For example, to determine intervals of confidence of 90%, centiles 5% and 95% must be used.

When N is an even number, the median is a mean value of 2 terms which intervals of confidence are not symmetrical, even if they are of same width: it is then needed to combine them to form the interval of confidence of the median. A Monte-Carlo determination of the standard deviation $\sigma_{Med,N}$ of the estimates of the median including odd numbers for N and their immediately following even number gave the results shown in Figure 14.

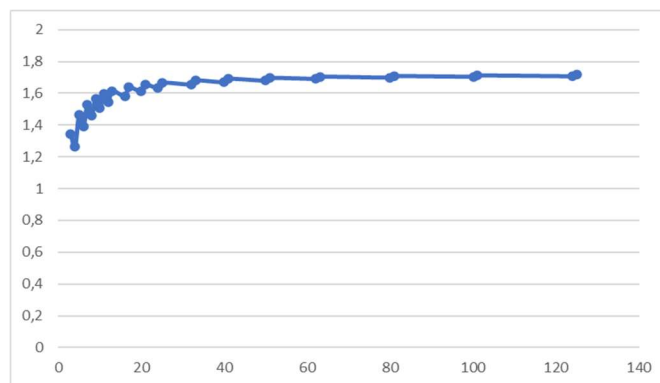


Figure 14: Ratios of standard deviations $\sigma_{Med,N}/\sigma_{Mean,N}$, where $\sigma_{Med,N}$ are the standard deviations of the estimates of the median and $\sigma_{Mean,N}$ are the standard deviations of the estimates of the mean values, as function of N .

It can be seen that:

- ✚ Intervals of confidence on median values is narrower when N is an even number than when N is an odd number;
- ✚ The difference between them is significant when N is low (example: -8,3% between $N=3$ and $N=4$), and is reduced when N increases until becoming almost insignificant.

Figure 15 displays the results of the equation $\frac{(\sigma_{Med,N} - \sigma_{Med,N+1}) \times N}{\sigma_{Med,N} + \sigma_{Med,N+1}}$, which was constructed to show the evolution of these differences in function of N.

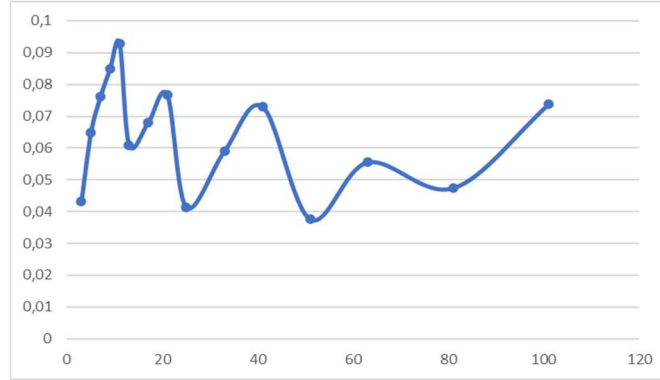


Figure 15: Values of $\frac{(\sigma_{Med,N} - \sigma_{Med,N+1}) \times N}{\sigma_{Med,N} + \sigma_{Med,N+1}}$ as function of N, where N is an odd number.

This curve suggests that, despite significant individual variations along N, the differences of intervals of confidence on the medians between N_{odd} and $N_{odd}+1$ is in the range of $\frac{0,06 \cdot \sigma_{Med,N}}{N}$.

Estimation of intervals of confidence on p_i for median values of i , using the ratio $\sigma_{Med,N}/\sigma_{Mean,N}$

Equation (26) has been established to provide approximations of $\sigma_{Med,N}/\sigma_{Mean,N}$ ratios to the nearest 0,01, for $N > 6$.

$$\frac{\sigma_{Med,N}}{\sigma_{Mean,N}} = 1,732 - \frac{1,45}{N} \quad (26)$$

where:

$\sigma_{Med,N}$ is the standard deviation of the estimates of the median of the median data among the N data (i.e. for $i = N/2$),

$\sigma_{Mean,N}$ is the standard deviation of the estimates of the mean of the median data among the N data (i.e. for $i = N/2$)

It can be noted from Equation (26) that $\sigma_{Med,N}/\sigma_{Mean,N} \rightarrow \sqrt{3}$ when $N \rightarrow \infty$, greater than 1,5 usually admitted for this ratio. As, the stand deviation of squared distribution of width equal to 1 is $1/2\sqrt{3}$, it follows that $\sigma_{Med,N} \rightarrow 1/2N$ when $N \rightarrow \infty$. However, as the reference distribution is squared, the z-values of Gaussian distribution cannot be used to compute intervals of confidence by enlarging the standard deviations, at least for low values of N. In addition to results of § 5.1.1, the p_i distributions were computed by the binary search method for a selection of i values for which $i = (N + 1)/2$ (median values of N). The corresponding results are provided in annex, Table A2.

It comes from these results that the Gaussian approximation provides limits of the IC intervals:

- ✚ To the nearest 0,01 when $N > 11$ for the 80% level of confidence;
- ✚ To the nearest 0,001 when $N > 50$ for the 80% level of confidence;
- ✚ To the nearest 0,01 when $N > 38$ for the 99% level of confidence;
- ✚ To the nearest 0,001 when $N > 180$ for the 99% level of confidence;

When those conditions are not fulfilled, the use of the Gaussian approximation provides a pessimistic determination of the limits of confidence (i.e. the real interval of confidence is better than the calculated one). In those cases, results in annex, Table A2, can be used to get a more accurate determination of it.

5.1.4 Conclusions concerning p_i distributions

The distribution of appropriate P_i values as function of i is very close to a straight line. Differences between true values of P_i and their linear approximations is less than 0,01 when usual equations are used, and less than 0,001 when an improved equation is used to determine “ a ” as a function of N . There is then no need to use more complicated functions of approximation (for example polynomial of higher degree) to find an adequate “ a ” value.

Usually recommended P_i values (see Equations (2) and (3)) are close to the median values of the p_i distributions.

The true median values that we computed are very close to straight lines that link P_1 and P_N . We then proposed two slightly more sophisticated equations than these classical ones (see Equations (5),(20),(22) and even more (23)), that produce results closer to the median values of p_i distributions.

When N is large (typically more than 30), the scatter on “ a ” is so huge that no care is needed for selecting an accurate value of “ a ”.

Anyway, the question of which central parameter of the p_i distributions is relevant cannot be solved at this stage and needs consideration to which distribution law is followed by the data to handle. As it will be seen further in § 5.2 and in § 5.3, forms of distributions of p_i and forms of distribution of the data interact so that values of “ a ” need to be determined for each specific case. When this interaction is unknown for the data distribution in question, choosing an equation that produces P_i values close to the median is a good solution, because the median is preserved during inverse transformations, and so the 'D-rankits' (rankits after inverse transformation in the 'D' distribution) thus produced are also median, and therefore acceptable in all cases, even if they are not fully optimised.

We determined an empirical polynomial of degree 2 (see Equation (25)) which produces approximate values of 5%-centiles of p_i distributions to the nearest 0,01, for $N \leq 100$. This can be used to determine intervals of confidence of probability plots, especially when the distribution is not normal.

We also determined an empirical equation (see Equation (26)) to determine IC on median values of p_i , what can be used to determine IC of median values of any distribution, as soon as its cumulative function is known.

5.2 z_i distributions for $N = 2$ to 30 for the Gaussian distribution

5.2.1 Introduction

Following § 2.1, § 2.4 and § 2.5, normal probability plots can be used to:

-  Check the normality of the distribution;
-  Detect possible outliers;
-  Estimate mean values and standard deviations using linear regressions.

Appropriate G-rankits for these 3 different scopes might be different.

For checking normality of the distribution, envelope curves corresponding to a fixed level of confidence are appropriate: they can be used to check whether a straight line can be drawn within these envelope curves. For detecting outliers, envelope curves are also relevant: using them enables to check whether the distribution can be regarded as Gaussian by suppressing a limited number of dots, as shown on Figure 3.

The ordinate at origin of the regression straight line is needed for estimating the mean value of a Gaussian distribution, while the slope is needed for estimating its standard deviation. These two issues are dealt with in the present § 5.2.

The Monte-Carlo method was used to determine the slope and the ordinate at origin (mean and median values) of the correlation lines between the above displayed z_i (plotted as “y” values) and P_i values (B-rankits) in accordance with § 3 (plotted as “x” values) as follows:

1. P_i defined according to Equation (1): $P_i = \frac{i-0,5}{N}$;
2. P_i defined according to Equation (2): $P_i = \frac{i-\frac{3}{8}}{N+\frac{1}{4}}$;
3. P_i defined according to Equation (3): $P_i = \frac{i-0,3175}{N+0,365}$, $P_N = 0,5\frac{1}{N}$, $P_1 = 1 - P_N$;
4. P_i defined according to Equation (19): $P_i = \frac{i-a}{N+1-2a}$ with $a = 0,5 \cdot \left(N + 1 - \frac{N-1}{2 \times 0,5\frac{1}{N}-1} \right)$;
5. P_i defined according to Equation (22): $P_i = \frac{i-a}{N+1-2a}$ with $a = 0,5 \cdot \left(N + 1 - \frac{N-1}{\left(2 \times 0,5\frac{1}{N}-1 \right) \left(1 + \frac{1}{20N+1} \right)} \right)$;
6. P_i defined according to Equation (23): $P_i = \frac{i-a}{N+1-2a}$ with $a = 0,33 - \frac{1}{3,7 \cdot N + 18,2}$;
7. P_i defined according to Equation (12): $P_i = \frac{i}{N+1}$, (mean value of p_i);
8. P_i defined as median value of p_i (see § 5.1.1 and values of tables A1 in annex);
9. P_i defined as p_i corresponding to Z_i , the mean value of z_i , as determined at § 5.2.3;
10. P_i defined according to Equation (27): $P_i = \frac{i-a}{N+1-2a}$ with $a = 0,41 - \frac{1}{1,5 \cdot N + 10}$.

A 11th possibility would have been to define P_i as the median value of z_i , as determined here upper. However, following the statements of § 3.5 and Figure 8, there is no need to determine them by using the Monte-Carlo method, as they can be determined directly from results of § 5.1.1, using the reverse Gaussian distribution law.

To simplify the calculations, z_i were in fact plotted as “x” values and P_i values (B-rankits) were in fact plotted as “y” values, so that the slope of the regression line and the ordinate at origin directly provides the expected results.

5.2.2 Distributions of P_i (B-rankits) corresponding to Z_i (G-rankits), mean value of z_i

The Monte-Carlo method was used to determine the mean values and standard deviations of z_i as function of i and N , for the Gaussian distribution. These values were determined to the nearest 0,001. Detailed results are provided in annex, Table A3.1 and Table A3.2. Figure 16 shows a selection of them for $N = 3, 5, 8, 13, 20$ and 30 .

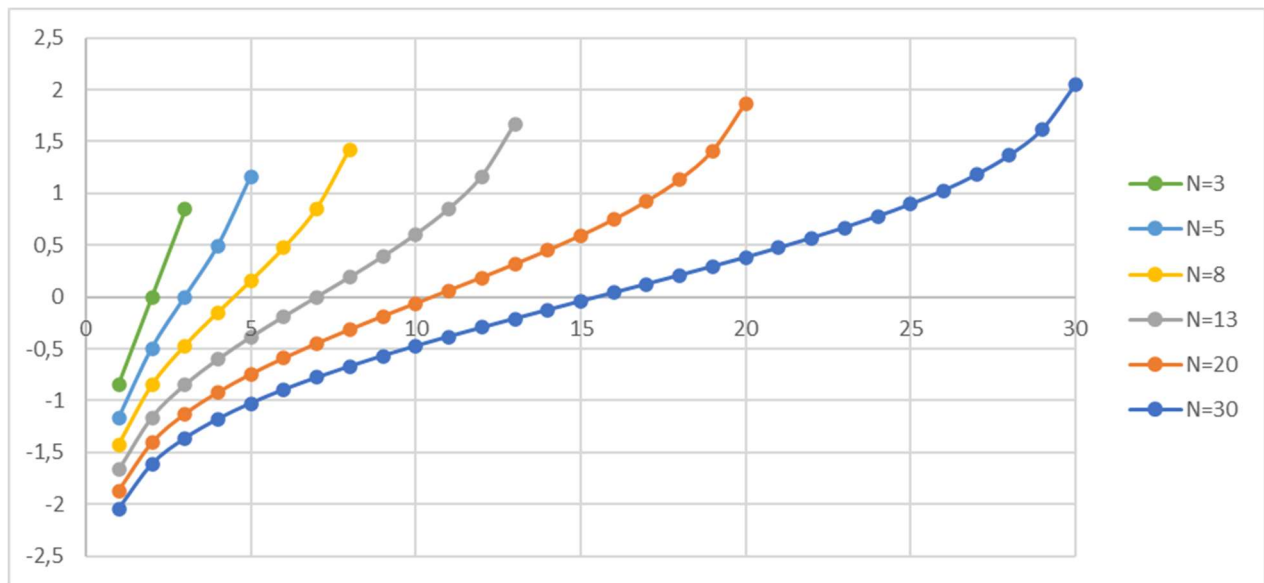


Figure 16: Distributions of Z_i (G-rankits of Gaussian distributions) as functions of i , for various values of N .

Obviously, corresponding P_i can be determined from Z_i by using the inverse normal distribution law.

5.2.3 Determination of "a" with Monte-Carlo method

The determination "a" was also performed using regression parameters of Z_i . To do so, series of z_i values were randomly created and ordered. Regression lines of corresponding p_i were determined. "a" values for these regression lines were then determined. This enables to determine overall mean values of "a", as a function of N . Table 5 and Figure 17 provide the results of these determinations. Note that for $N > 250$, $2u$ becomes huge and corresponding dots are not displayed on Figure 17.

Table 5. Mean values of "a" of Equation (5) obtained by linear regression for the Gaussian distribution, as a function of N .

N	"a"	2u	N	"a"	2u	N	"a"	2u
2	0,33005	0,00023	19	0,38447	0,00050	63	0,40073	0,00254
3	0,34060	0,00024	20	0,38474	0,00053	80	0,40312	0,00322
4	0,34805	0,00027	21	0,38595	0,00070	100	0,40469	0,00400
5	0,35413	0,00031	22	0,38671	0,00073	125	0,40662	0,00499
6	0,35873	0,00034	23	0,38741	0,00076	160	0,40765	0,00640
7	0,36301	0,00038	24	0,38819	0,00079	200	0,40876	0,00800
8	0,36631	0,00042	25	0,38840	0,00115	250	0,41004	0,00998
9	0,36891	0,00046	26	0,39020	0,00094	320	0,40980	0,00828
10	0,37153	0,00050	27	0,39076	0,00098	400	0,41066	0,00537
11	0,37321	0,00047	28	0,39143	0,00101	500	0,40268	0,00672
12	0,37542	0,00052	29	0,39177	0,00105	630	0,40136	0,01004
13	0,37700	0,00050	30	0,39156	0,00098	800	0,41227	0,01279
14	0,37875	0,00048	31	0,39181	0,00098	1000	0,41259	0,01593
15	0,37953	0,00050	32	0,39272	0,00104	1250	0,41247	0,01992
16	0,38120	0,00051	40	0,39579	0,00162	1600	0,41336	0,02558
17	0,38208	0,00051	50	0,39857	0,00201	2500	0,41396	0,03983
18	0,38310	0,00048						

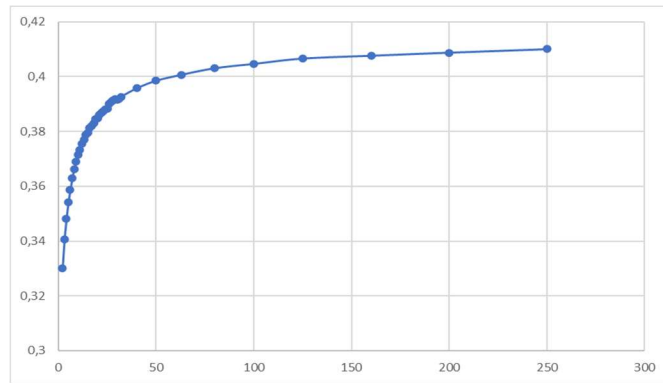


Figure 17: Mean values of “a” of Equation (5) obtained by linear regression for the Gaussian distribution, as a function of N.

For $N > 250$:

- ✚ Mean values of “a” tend to be 0,41;
- ✚ Amounts of calculation for getting accurate determinations becomes huge, and there is no need for them as, in these situations, $a \ll N + 1 - 2a$.

Equation (27) appeared to provide “a” values as a function of N with enough accuracy compared to $N + 1 - 2a$.

$$a = 0,41 - \frac{1}{1,5.N + 10} \quad (27)$$

Where: “N” is the total number of values of the series.

In addition to mean values, this method provides standard deviations of “a”. This is interesting because it provides an information about the scatter with which “a” can vary around its mean value. Figure 18 displays the results of it.

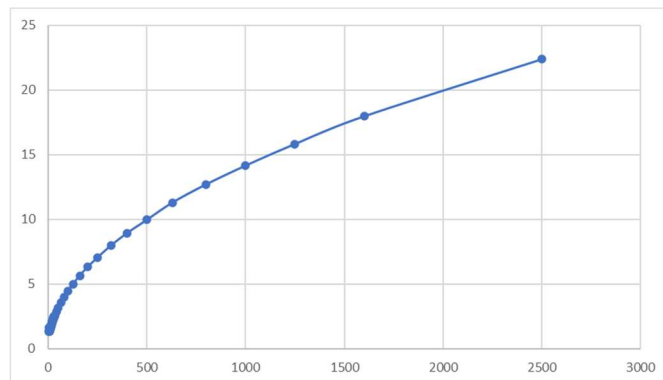


Figure 18: Standard deviations of “a” of Equation (5) obtained by regression for the Gaussian distribution, as a function of N.

Equation (28) appeared to provide a correct approximation of standard deviations of “a” values as a function of N, for $N > 10$.

$$\sigma_a = \sqrt{0,2.N + 0,4} \quad (28)$$

Where: σ_a is the standard deviation of estimates of “a”
 “N” is the total number of values of the series.

In all cases, $\sigma_a > 1,35$, which means that the width of IC on “a” is always more than $\pm 2,7$, what is huge compared to differences between recommended values for it (see § 2).

5.2.4 Comparison of methods to determine the ordinates at origin of the normal plotting curves (determination of the mean value)

Average values and standard deviations of ordinate at origin

The average value and the standard deviation of ordinates at origin of the regression straight lines using G-rankits as function of N were determined using the Monte-Carlo.

These determinations showed that:

- ✚ No significant differences exist between the different options listed in 5.2.1 to determine the used G-rankits. In other words, extremal dots on the normal probability curve are of low influence on the determination of ordinates at origin of regression straight lines;
- ✚ In the same way, no bias and no difference in the standard deviations of results apply whatever these different options;
- ✚ The standard deviation of ordinates at origin is linked to N by Equation (29).

$$\sigma_{m,N} = \frac{\sigma_{pop}}{\sqrt{N}} \quad (29)$$

Where:

$\sigma_{m,N}$ is the standard deviation of estimation of the ordinate at origin of the regression straight line, i.e. of the mean value,

σ_{pop} is the standard deviation of the whole population of the data

"N" is the total number of data of the series.

As conclusion, the determination of a mean value using regression straight lines is as accurate as the habitual method, whatever the way of computing the G-rankits (the computation of an ordinate at origin of a regression straight is actually a computation of a mean value, under a different form).

5.2.5 Comparison of methods to determine the slopes of the normal plotting curves (determination of the standard deviation)

Average values of slope coefficients

A value equal to 1 demonstrates that no bias is existing between the average slope and the standard deviation when a normal probability plot is used as shown in Figure 4.

The standard deviation of all these slopes were also determined, to get an idea on how scattered the results of slope coefficients are, independently to their possible bias. This makes sense in the perspective of using them to estimate the standard deviations of Gaussian distributions, as shown in Figure 4.

It was also verified that ordinates at origin of all those correlation lines are not significantly different from 0, by the mean of a statistical test using the interval of confidence of the corresponding determinations.

Figure 19.1 to Figure 19.10 here after display the results of these determinations.

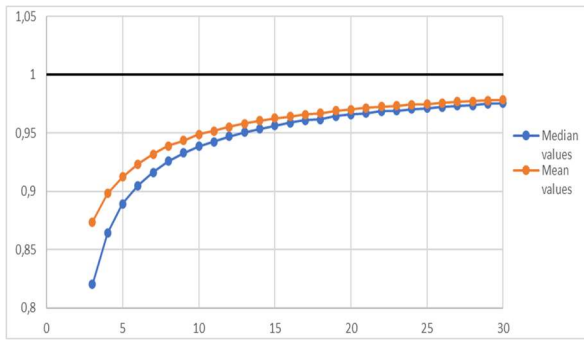


Figure 19.1. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as $P_i = \frac{i-0.5}{N}$.

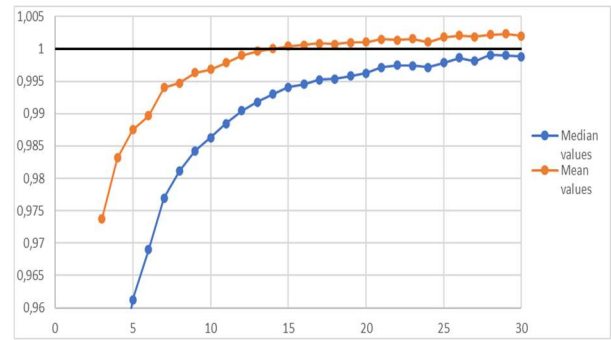


Figure 19.2. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as $P_i = \frac{(i - \frac{3}{8})}{(N + \frac{1}{4})}$.

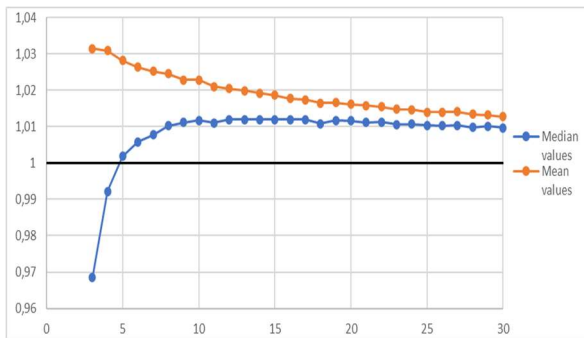


Figure 19.3. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as $P_i = \frac{i-0.3175}{N+0.365}$, $P_N = 0.5$, $P_1 = 1 - P_N$.

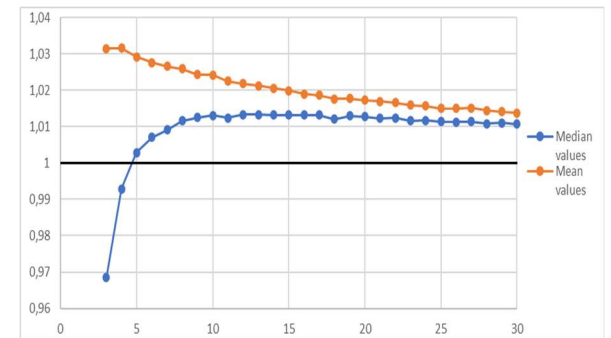


Figure 19.4. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as $P_i = \frac{i-a}{N+1-2a}$ with $a = 0.5$, $(N + 1 - (N - 1)/(2 \times 0.5^N - 1))$.

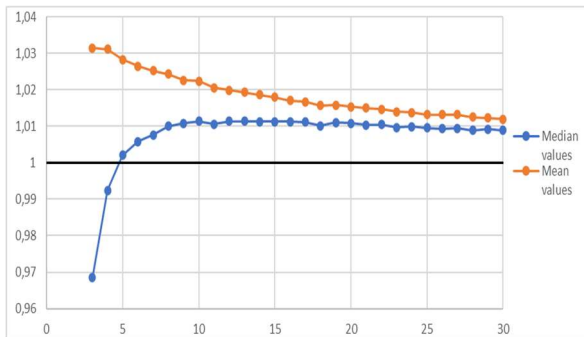


Figure 19.5. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as $P_i = \frac{i-a}{N+1-2a}$ with $a = 0.5$, $(N + 1 - (N - 1)/(2 \times 0.5^N - 1)) \left(1 + \frac{1}{20N+100}\right)$.

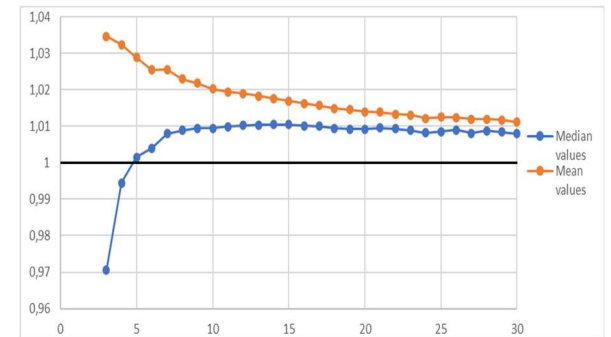


Figure 19.6. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as $P_i = \frac{i-a}{N+1-2a}$ with $a = 0.33 - 1/(3.7 \cdot N + 18.2)$.

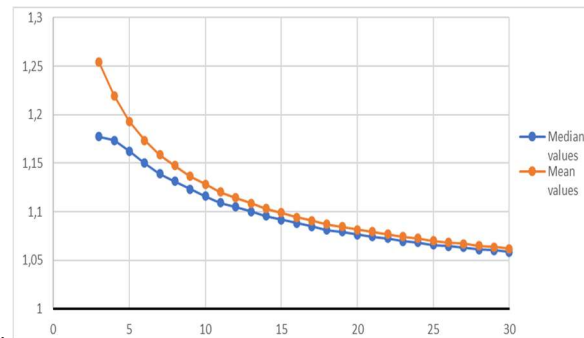


Figure 19.7. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as $P_i = \frac{i}{N+1}$, (mean value of p_i).

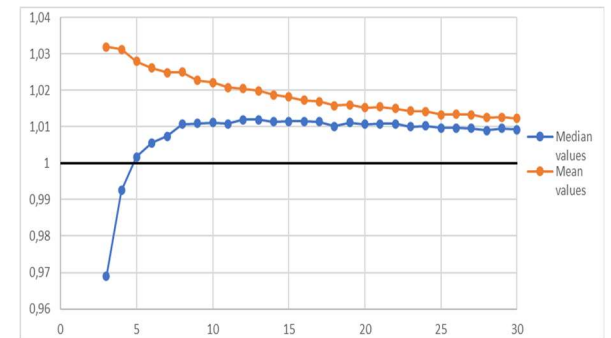


Figure 19.8. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as median value of p_i (see § 5.1.1 and values in Table A3.1 in annex)

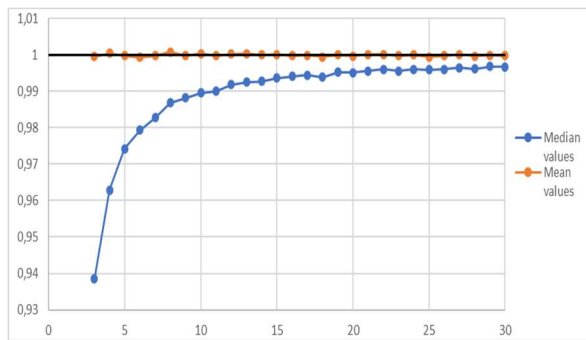


Figure 19.9. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as the p_i corresponding to Z_i , the mean values of z_i .

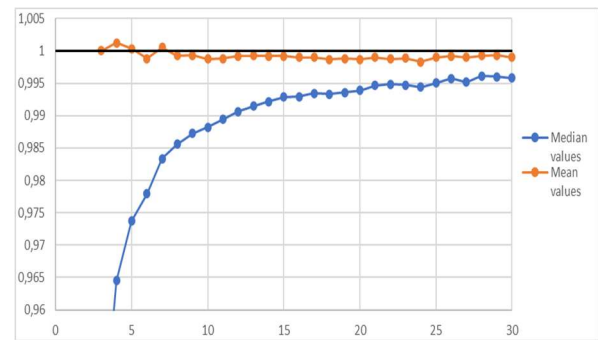


Figure 19.10. Mean values and median values of slopes of regression lines as functions of N , using P_i defined as $P_i = \frac{i-a}{N+1-2a}$ with $a = 0,41 - 1/(1,5 \cdot N + 10)$.

Conclusions concerning the biases:

Some slight irregularities are visible on some figures. They are very likely to come from uncertainties on determinations of slopes. Erasing them would request an important increase in the volumes of calculation, and it was not decided to go in that direction.

For all options, significant differences apply between the use of median values and the mean values to characterise the average value of slopes. This shows that distributions of slopes are not symmetrical, and this asymmetry depends on the option used to determine the applied P_i (some options lead medians larger than mean values and some others lead to the reverse situation, some options lead medians closer to 1 while some others lead to the reverse situation).

Options 3, 4, 5 and 6 behave very similarly to option 8, which is the pure median one. This is consistent with the facts that Equation (3), Equations (5) + (20) and Equations (5) + (22) were designed to produce P_i values approximating the median of the p_i distributions and confirms the conclusions of § 5.1.4. These options seem to lead to a bias (+0,01 in most cases), when N is increasing. That is contradictory with the evidence that all modes of determination of P_i converge to same values when $N \rightarrow \infty$. We then performed some verifications that confirmed that all modes lead to absence of bias when $N \rightarrow \infty$, as shown in Table 6:

Table 6. Average slopes obtained with Equation (3) for large values of N .

N	50	100	315	1000
Mean Value	1,009	1,003	1,0018	1,0007
IC ($k=2$)	$\pm 0,0045$	$\pm 0,003$	$\pm 0,0018$	$\pm 0,0010$

The results of this table confirm that the average slopes of correlation lines converge to 1 when $N \rightarrow \infty$, even if large values of N (typically $N=1000$) need to be used to reach the nearest 0,001 for them.

Options 9 and 10 are the most performant. Option 9 request to use P_i of Tables in Annex (determined by the Monte-Carlo method), but option is easy to implement, only using Equations (5) + (23).

Option 2 ($a = 0,3175$) seems to behave closely to option 8 when N is large enough, leading us to suppose that Equation (2) was intended to provide P_i values that produce Z_i close to the ones that we determined by the Monte-Carlo method, and used for option 9.

Options 1 ($a = 0,5$) and 7 ($a = 0$) are those which show the slowest convergence to 1 when N increases.

Scatter linked to the determination of these slopes

Beside the difference between the true standard deviation of a Gaussian distribution and its average estimate using the slope of the regression lines, the performance of these estimations is also impacted by the scatter around the average value, as a function of the number N and of the option used to determine P_i .

To avoid too heavy calculations, the standard deviation of slopes was chosen as the unique parameter to characterise this scatter. This is enough to compare properly the 10 options. However, as distributions of z_i are obviously asymmetrical (1- Differences are shown between mean and median values and 2- standard deviations are significant when compared to the averages), these standard deviations cannot be used to determine accurate intervals of confidence on the z_i (limits of IC are not symmetrical around the central value). For determining them, centiles of distributions need to be determined, and this can only be performed by using the Monte-Carlo method, what implies huge calculations.

Results are shown on Figure 20. The number of repetitions used enabled to determine these standard deviations to the nearest 0,002 for $N \leq 4$ and to the nearest 0,001 for $N \geq 5$.

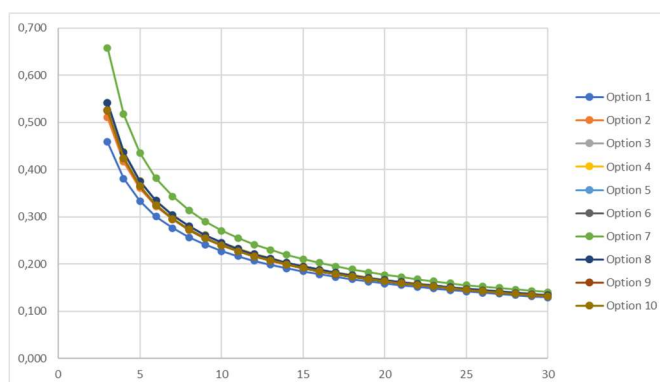


Figure 20: Standard deviations of the estimates of the data standard deviation, for the Gaussian distribution, as function of N and option used to determine P_i , for $N \leq 30$.

An attempt not displayed in here showed that these curves become close to straight lines when a log transformation is applied to both axes.

Conclusions concerning scatter:

At first glance on Figure 20, options 2, 3, 4, 5, 8, 9 and 10 seem to show quite similar scatter, while option 1 is significantly better and option 7 is significantly worse for small values of N . But a look at Figure 19.1 to Figure 19.10 shows that significant bias correction factors need to be applied when using these options 1 and 7, so that effective scatter on estimations after corrections of bias is not significantly different whatever the option.

5.2.6 Intervals of confidence on z_i

5.2.6.1 Validity of statements of § 3.5

The validity of the statements of § 3.5 has been verified by comparing results obtained with the method of § 3.5 and the results obtained by the Monte-Carlo method (see § 5.1.1). This check has been performed on the case of normal probability plots, for the 2,5% centile (lower limit of the interval of confidence of 95%).

Figure 21 shows the differences in the results between these two ways of calculation.

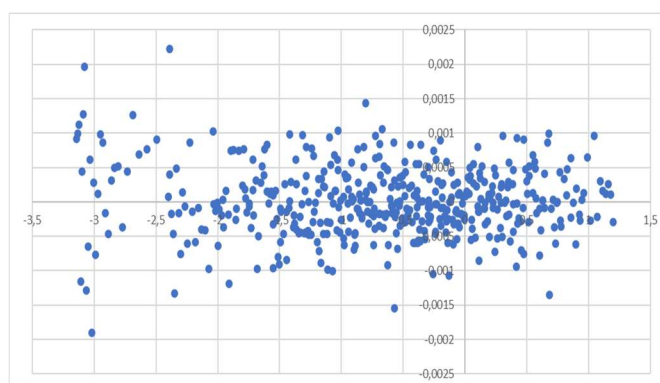


Figure 21. Differences between the centile 2,5% computed with B-rankits and with Z_i rankits, as function of the true value of the centile, for $3 \leq N \leq 30$ and $1 \leq i \leq N$

It can be seen that no significant difference can be seen between the two series of results, validating the calculation by using the B-rankits (the max differences are within the accuracy of the determinations, as stated in § 5.2.2).

5.2.6.2 Limits of intervals of confidence

Figure 22 and Figure 23 show the results of limits of the intervals of confidence of 95% for z_i , for $N=5$, $N=9$, $N=15$ and $N=28$ as function of i and as function of Z_i . It must be noted that, for Figure 23, a choice must be made among the different ways to determine the B-rankits to be used for the computation of Z_i . To do so, we have chosen to use the Equation (2), which is the most classical one. As seen before, differences between Equations (2) and (3) are low.

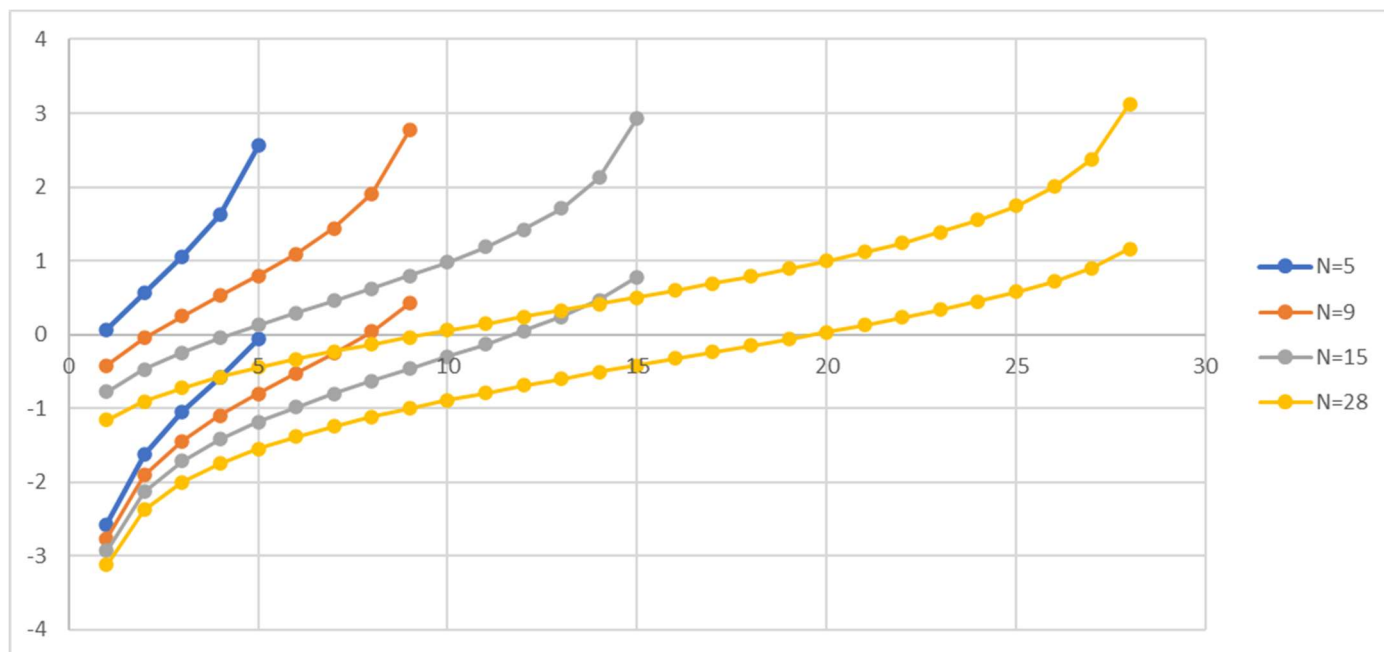


Figure 22. Limits of intervals of confidence of 95% of z_i , as function of the i for $N = 5$, $N = 9$, $N = 15$ and $N = 28$

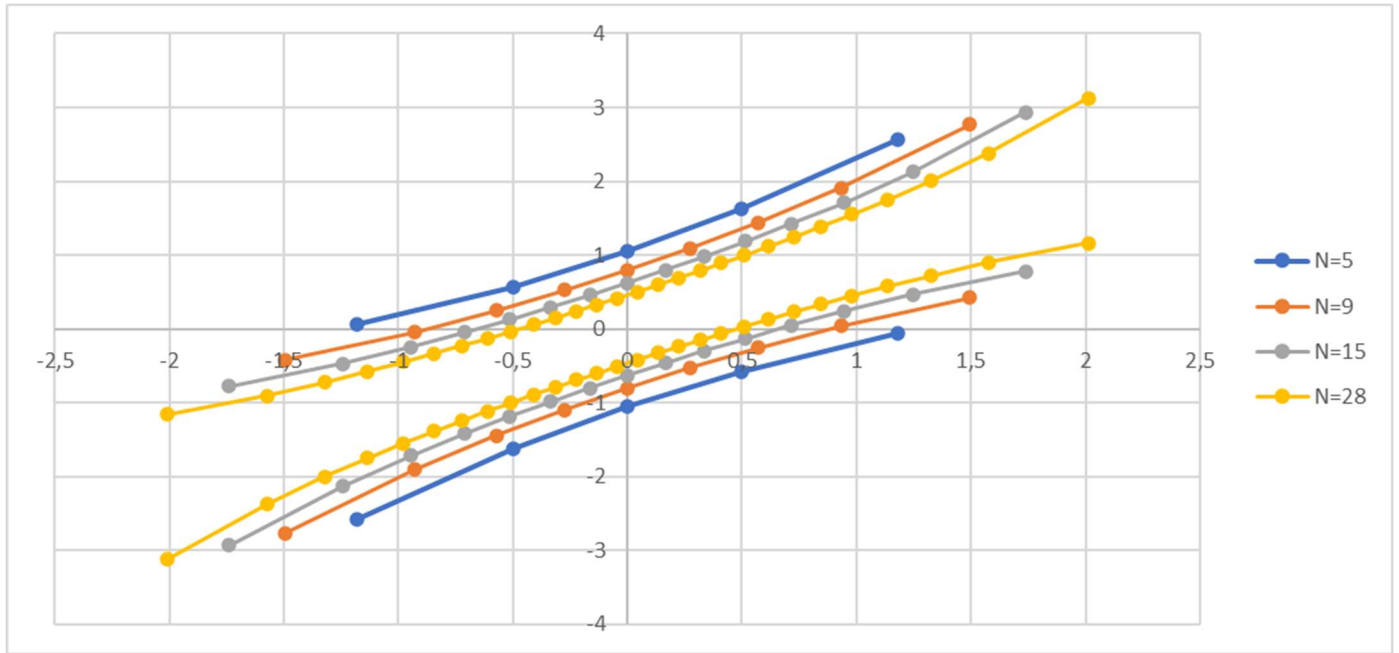


Figure 23. Limits of intervals of confidence of 95% of z_i , as function of the z_i for $N = 5$, $N = 9$, $N = 15$ and $N = 28$

Figure 23 shows that intervals of confidence have got hyperbolic shapes, as expected when they are determined by using calculations of uncertainties on coefficients of correlation.

Note that, in practical cases, the true values of the mean μ and standard deviation σ are unknown. Only their estimates are known. However, knowledge of these values μ and σ is necessary to plot Figure 22 and Figure 23 and use them to check whether a set of data falls within the corresponding confidence intervals. A future study is planned to determine techniques for overcoming this problem.

5.2.6.3 Comparison of these results with the intervals of confidence computed from uncertainties on coefficients of correlation

When occurrences are independent and follow a Gaussian distribution, Equation (30) can be used to describe the distribution of residues.

$$\bar{y}_j - t \cdot \sigma \cdot \sqrt{\frac{1}{N} + \frac{(x_j - \bar{x})^2}{\sum (x_i - \bar{x})^2}} \leq y_j \leq \bar{y}_j + t \cdot \sigma \cdot \sqrt{\frac{1}{N} + \frac{(x_j - \bar{x})^2}{\sum (x_i - \bar{x})^2}} \quad (30)$$

where t is the Student value corresponding to the level of confidence of the interval

σ is the standard deviation of the distribution

N is the number of couples of values integrated in the regression

x_i is the i^{th} data and y_j is an expected value of the j^{th} data.

In our case, $\sigma = 1$ (case of the centred reduced normal distribution).

Results found in § 5.2.6.2 and determined with Equation (30) were compared. To do so, Figure 24 plots:

- ✚ the quantity $IC_{Reg} = \sqrt{\frac{1}{N} + \frac{(x_j - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$ (IC determined with Equation (30)) as ordinate;
- ✚ as a function of $IC_{MC} = \frac{y_{j.MC.0,975} - y_{j.MC.0,025}}{2 \times 1,96}$ (IC determined with the Monte-Carlo method).

This way of defining IC_{MC} , without reference to the central value, avoids the problem of defining the adequate B-rankit for this central value.

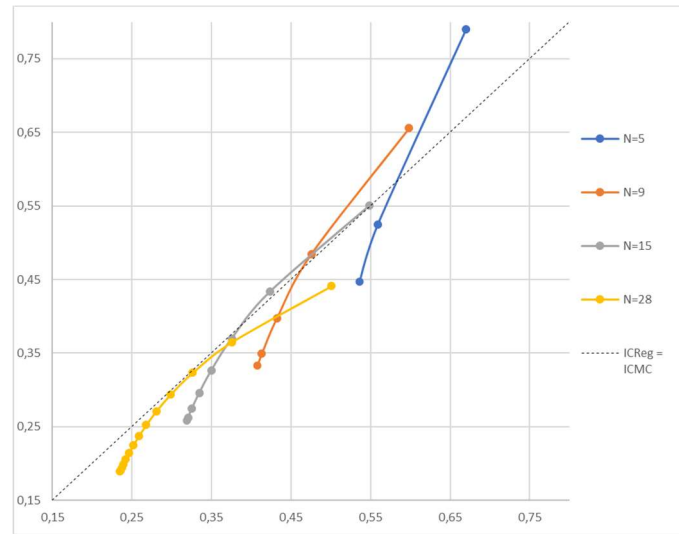


Figure 24. Standard deviations of intervals of confidence of z_i "IC_{Reg}" computed with Equation (30) as function of corresponding values "IC_{MC}" determined with the Monte-Carlo method, for $N = 5$, $N = 9$, $N = 15$ and $N = 28$

Figure 24 demonstrates that Equation (30) is an approximation of the true function.

The bottoms of the curves correspond to the central values (median value when N is an odd integer). For these values, the widths of the IC computed with Equation (30) are somewhat underestimated (about 20%). This is consistent with:

- ✚ Results of Equation (29) at § 5.3.2 (concerning the scatter of ordinates at origin of regression lines);
- ✚ And results of § 5.2.6.2 (concerning a comparison between scatters of results obtained by interpolation and by regression).

5.2.7 Conclusions concerning z_i distributions for the Gaussian distribution

Conclusions concerning the determination of the mean of the represented data:

Determining a mean value using regression lines is as accurate as the usual method, regardless of how the G-rankits are calculated.

Conclusions regarding the determination of the standard deviation of the represented data:

For all the options evaluated in this study (see § 5.2.1), significant differences apply between the use of median values and mean values to characterise the mean value of the slopes of the regression lines, related to the standard deviation of the represented distribution. This is due to the asymmetry of the distribution of standard deviation estimates, particularly when N is small.

The options which correspond more or less to a selection of P_i values close to the median ($\alpha = 0,3175$, $\alpha = 0,375$, $\alpha = 0,5$, $\alpha = 0,5 \cdot \left(N + 1 - (N - 1) / (2 \times 0,5^{\frac{1}{N}} - 1) \right)$, $\alpha = 0,5 \cdot \left(N + 1 - (N - 1) / (2 \times 0,5^{\frac{1}{N}} - 1) (1 + 1 / (20N + 100)) \right)$ and $\alpha = 0,33 - 1 / (3,7 \cdot N + 18,2)$) seem to lead to a bias (+0,01 in most cases), even when N is large (up to 1000). However, some verifications have confirmed that all modes lead to the absence of bias when $N \rightarrow \infty$.

The equation $\alpha = 0,41 - 1 / (1,5 \cdot N + 10)$ has been determined empirically to produce values of P_i that avoid bias, even when N is small. We can conclude from this equation that $\alpha \rightarrow 0,41$ when $N \rightarrow \infty$.

The options $a = 0$ and $a = 0,5$ lead to significant biases for the determination of standard deviations.

There is no significant difference between the different options with regard to the dispersion of the estimates of the standard deviations of the data represented.

Conclusions regarding the determination of the confidence envelope curves for the represented data:

Regarding the confidence envelope curves of normal probability plots, we found that the use of the centiles of the p_i distributions provides accurate determinations, while the classical IC equation for the regression curves appears to be an approximation. However, checking whether the data set falls within the corresponding confidence intervals is hampered by the fact that the true values of the mean μ of the standard deviation σ are unknown. A future study is planned to identify techniques to overcome this problem.

5.3 Determination of the zr_i distributions for $N = 2$ to 30 for the distributions of standard deviation estimates of a Gaussian distributed population

5.3.1 Introduction

Following § 2.1, § 2.4 and § 2.5, normal probability plots can be used to check the normality of the distribution, detect outlier and/or estimate mean values and standard deviations using linear regressions.

This can in fact be carried out for any distribution of probabilities, using the corresponding transformation of variable. Equations (7) and (8) provide this for the case of the estimation of a standard deviation from limited series of values. Figure 6 shows an example of “SD estimating probability plot”, that enables to:

-  Check whether the observed results follow the expected distribution;
-  Detect possible outlier;
-  Estimate the corresponding standard deviation.

Check whether the observed results follow the expected distribution:

In the same way that normal probability plots can be used to check whether a population is normally distributed, SD probability plots show whether a population of SD results distributes as it is expected for them (i.e. against Equation (8)). If not, it can mean that:

-  One or several outliers are present. Then, the suppression of them enables the results to align on the expected straight line;
-  The SD estimates do not pertain to a same population, i.e., the hypothesis of homoscedasticity cannot be accepted for that set of estimates of a SD.

These statements were not developed in this study. In particular, the way in which the plots discard from the straight line is probably full of information about the reasons why the homoscedasticity hypothesis cannot be accepted. However, this issue is worth an extensive study and needs to be treated in a separate work.

Detect possible outlier:

It is well known that outliers are strongly impacting the determination of standard deviations, even more than for the determination of mean values. For this reason, many methods were developed to detect them and eliminate their deleterious effects, see [2].

Standard deviation probability plots enable to visualise their existence and even to use methods that were initially developed only for Gaussian distributed populations. Centiles of p_i distributions (see § 3.5) can be used to determine limits outside of which the values may be regarded as outliers.

Estimate a standard deviation:

In the same way than for the normal probability plots, the abscissa where the straight line cuts the ordinate “1” is a good estimate of the true standard deviation of the population. Contrarily to the normal probability plot, the slope of the straight line is of no signification. Normal distributions request 2 parameters (mean and standard deviation) to describe them, while Equation (8) contents only one: “ σ ”.

In fact, in Equation (8), ordinate “1” and slope are theoretically linked, as the point (0;0) is always supposed to be part of the straight line. A significant deviation to this tends to mean that the hypothesis of homoscedasticity is not fulfilled.

As for normal probability plots, appropriate B-rankits for these 3 different scopes might be different. However, this study is focused on the last one, because it is a potential robust method of estimating standard deviations.

The Monte-Carlo method was used to determine the abscissa where the straight line cuts the ordinate “1” of the correlation lines between the above displayed zr_i (plotted as “y” values) and P_i values (B-rankits) in accordance with § 3 (plotted as “x” values) in 9 of the 10 options as detailed in § 5.2.1. Obviously, option 10, that refers to Equation (31) specially developed for the case of the Gaussian distribution is replaced by the following (see § 5.3.3):

10. P_i defined according to Equation (31): $P_i = \frac{i-a}{N+1-2a}$ with $a = a_{Nr} - \frac{1}{A_{Nr} \cdot N_s + B_{Nr}}$ (see § 5.3.3 for coefficients A_{Nr} and B_{Nr}).

To simplify the calculations, zr_i were in fact plotted as “x” values and P_i values (B-rankits) were in fact plotted as “y” values, so that the ordinate of the abscissa “1” directly provides the expected result.

5.3.2 Distributions of p_i corresponding to zr_i

Quadratic mean values, Mean values, Median values, 2,5% centile values, 97,5% centile values of zr_i for S-distributions as function of N_s (number of series), N_r (number of repetitions) and i were determined by the Monte-Carlo method for $N_s \leq 30$. Results are provided in annex, Table A4.

As an example, Figure 25 shows how zr_i computed with quadratic means distribute for a selection of N_s (number of series from 5 to 30) and a selection of N_r (number of repetitions from 2 to 25). For example, for $N_s=30$ and $N_r=2$, zr_i rank from 0,06 to 2,37, what correspond to min and max ordinates of the corresponding curves.

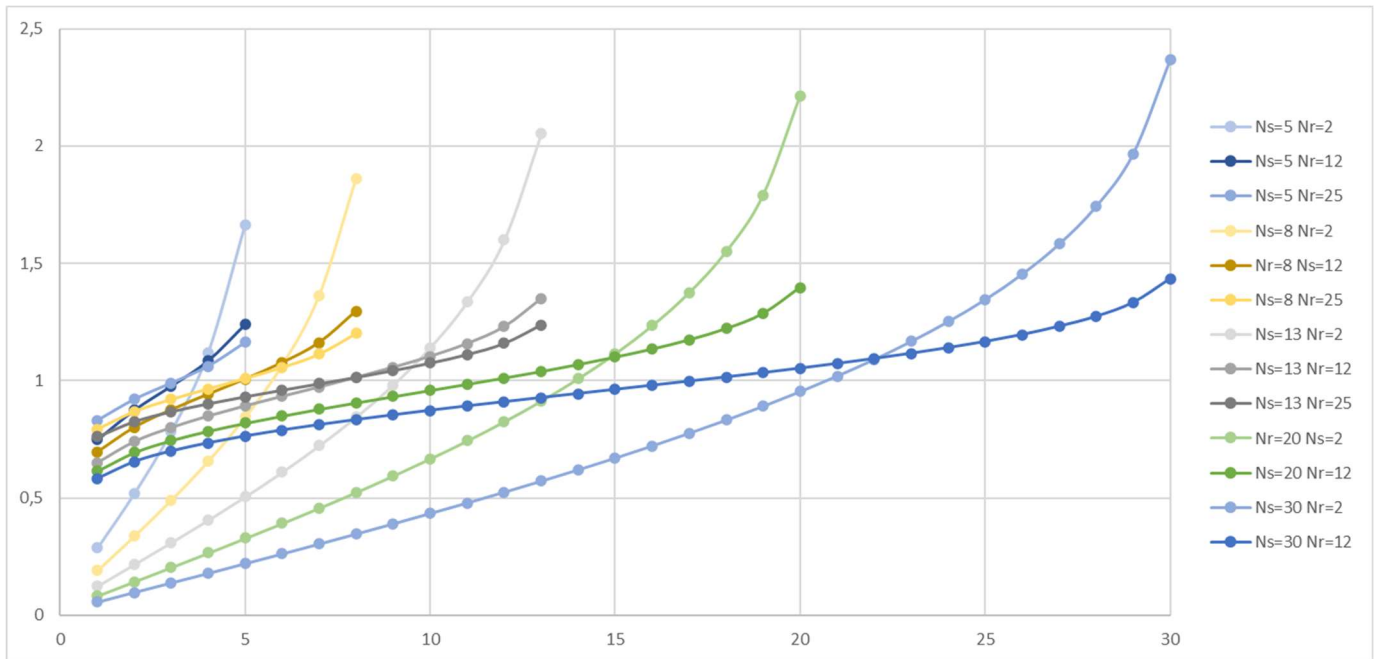


Figure 25. Values of ZR_i computed with quadratic means values, for a selection of N_s (number of series) and of N_r (number of repetitions), as a function of i .

Some features can be seen from Figure 25:

- ✚ The gap between ZR_N and ZR_1 increases when N increases;
- ✚ The i^{th} value for which the curve cuts the ordinate “1” is always greater than $\frac{N_r}{2}$ but tends to $\frac{N_r}{2}$ when N_s and/or N_r increase;
- ✚ The greater N_s and N_r , the more the curve is horizontal, the better is the accuracy in the determination of the intercept of abscissa “1” is, the better is the estimation of s .

5.3.3 Determination of “a” with Monte-Carlo method

In the same way than in § 5.1.2, the determination “a” was also performed using regression parameters of P_i . Table 7 and Figure 26 provide the results of these determinations.

Table 7. Mean values of “a” of Equation (5) obtained by regression for the SD distribution, as a function of Nr (number of repetitions) and Ns (number of series). (2u is the IC95% and SD is the standard deviation).

Nr=2				Nr=3				Nr=4				Nr=5			
Ns	a	2u	SD	Ns	a	2u	SD	Ns	a	2u	SD	Ns	a	2u	SD
5	0,0498	0,0008	2,7	5	0,2131	0,0009	2,7	5	0,2641	0,0006	3,1	5	0,2877	0,0006	3,2
6	0,0492	0,0009	2,7	6	0,2161	0,0009	2,7	6	0,2683	0,0007	3,1	6	0,2927	0,0007	3,3
7	0,0472	0,0007	2,8	7	0,2192	0,0007	2,8	7	0,2715	0,0007	3,1	7	0,2965	0,0008	3,4
8	0,0463	0,0007	2,8	8	0,2206	0,0007	2,9	8	0,2748	0,0008	3,3	8	0,2991	0,0009	3,5
9	0,0449	0,0007	2,8	9	0,2223	0,0008	3,0	9	0,2778	0,0009	3,3	9	0,3020	0,0009	3,5
10	0,0446	0,0008	2,9	10	0,2245	0,0009	3,1	10	0,2786	0,0009	3,3	10	0,3040	0,0010	3,6
13	0,0426	0,0010	3,0	13	0,2280	0,0011	3,3	13	0,2831	0,0012	3,6	13	0,3103	0,0013	4,0
16	0,0400	0,0012	3,3	16	0,2297	0,0013	3,6	16	0,2866	0,0014	3,9	16	0,3124	0,0016	4,5
20	0,0383	0,0013	3,3	20	0,2340	0,0015	3,8	20	0,2899	0,0017	4,3	20	0,3169	0,0019	4,8
25	0,0375	0,0017	3,8	25	0,2350	0,0019	4,2	25	0,2932	0,0022	4,9	25	0,3233	0,0024	5,3
32	0,0354	0,0020	4,0	32	0,2379	0,0023	4,6	32	0,2970	0,0027	5,3	32	0,3222	0,0030	6,0
40	0,0317	0,0025	4,4	40	0,2403	0,0029	5,2	40	0,2982	0,0033	5,8	40	0,3228	0,0036	6,4
50	0,0308	0,0031	4,9	50	0,2368	0,0036	5,7	50	0,3026	0,0041	6,4	50	0,3319	0,0046	7,3
63	0,0306	0,0038	5,3	63	0,2445	0,0045	6,4	63	0,3042	0,0053	7,5	63	0,3252	0,0056	7,9

Nr=6				Nr=7				Nr=8				Nr=9			
Ns	a	2u	SD	Ns	a	2u	SD	Ns	a	2u	SD	Ns	a	2u	SD
6	0,3076	0,0012	3,6	6	0,3168	0,0008	3,7	6	0,3224	0,0009	3,9	6	0,3271	0,0015	4,2
7	0,3107	0,0010	3,7	7	0,3206	0,0009	3,9	7	0,3262	0,0010	4,1	7	0,3309	0,0012	4,4
8	0,3141	0,0010	3,8	8	0,3226	0,0010	4,1	8	0,3291	0,0011	4,4	8	0,3340	0,0012	4,6
9	0,3167	0,0011	3,9	9	0,3249	0,0011	4,2	9	0,3327	0,0012	4,4	9	0,3363	0,0013	4,7
10	0,3193	0,0011	4,0	10	0,3282	0,0012	4,3	10	0,3337	0,0013	4,6	10	0,3410	0,0014	4,9
13	0,3239	0,0015	4,6	13	0,3347	0,0015	4,6	13	0,3394	0,0016	5,0	13	0,3452	0,0017	5,4
16	0,3291	0,0017	4,8	16	0,3369	0,0018	5,1	16	0,3425	0,0019	5,4	16	0,3499	0,0021	5,8
20	0,3298	0,0021	5,2	20	0,3430	0,0023	5,7	20	0,3474	0,0024	6,0	20	0,3544	0,0025	6,2
25	0,3342	0,0025	5,7	25	0,3461	0,0028	6,2	25	0,3499	0,0029	6,4	25	0,3531	0,0031	6,9
32	0,3379	0,0032	6,4	32	0,3460	0,0035	6,9	32	0,3538	0,0037	7,4	32	0,3579	0,0040	7,9
40	0,3436	0,0040	7,1	40	0,3507	0,0042	7,4	40	0,3550	0,0047	8,3	40	0,3612	0,0049	8,6
50	0,3433	0,0050	7,9	50	0,3524	0,0055	8,6	50	0,3583	0,0058	9,1	50	0,3621	0,0063	9,9
63	0,3432	0,0062	8,7	63	0,3491	0,0068	9,6	63	0,3625	0,0072	10,1	63	0,3624	0,0076	10,7

Nr=12				Nr=16				Nr=20				Nr=25			
Ns	a	2u	SD	Ns	a	2u	SD	Ns	a	2u	SD	Ns	a	2u	SD
6	0,3350	0,0017	5,0	6	0,3418	0,0020	5,8	6	0,3441	0,0022	6,3	6	0,3483	0,0024	6,8
7	0,3394	0,0014	5,1	7	0,3454	0,0016	5,9	7	0,3478	0,0017	6,5	7	0,3526	0,0019	7,3
8	0,3431	0,0013	5,2	8	0,3490	0,0015	6,0	8	0,3508	0,0017	6,8	8	0,3550	0,0019	7,5
9	0,3472	0,0014	5,4	9	0,3515	0,0017	6,3	9	0,3539	0,0018	6,9	9	0,3567	0,0021	7,7
10	0,3495	0,0016	5,7	10	0,3526	0,0019	6,6	10	0,3553	0,0020	7,0	10	0,3609	0,0023	8,0
13	0,3535	0,0020	6,1	13	0,3600	0,0023	7,0	13	0,3639	0,0025	7,9	13	0,3628	0,0029	8,9
16	0,3569	0,0024	6,7	16	0,3616	0,0027	7,7	16	0,3647	0,0031	8,6	16	0,3644	0,0035	9,7
20	0,3597	0,0029	7,2	20	0,3657	0,0034	8,4	20	0,3668	0,0038	9,4	20	0,3710	0,0042	10,6
25	0,3642	0,0037	8,2	25	0,3645	0,0041	9,2	25	0,3690	0,0047	10,5	25	0,3780	0,0052	11,6
32	0,3607	0,0045	8,8	32	0,3740	0,0053	10,5	32	0,3724	0,0059	11,7	32	0,3700	0,0065	12,8
40	0,3689	0,0057	10,0	40	0,3734	0,0065	11,6	40	0,3756	0,0073	12,9	40	0,3741	0,0081	14,4
50	0,3700	0,0071	11,2	50	0,3706	0,0082	13,0	50	0,3805	0,0090	14,2	50	0,378	0,010	16,2
63	0,3708	0,0088	12,3	63	0,379	0,010	14,3	63	0,384	0,012	16,6	63	0,370	0,013	17,7

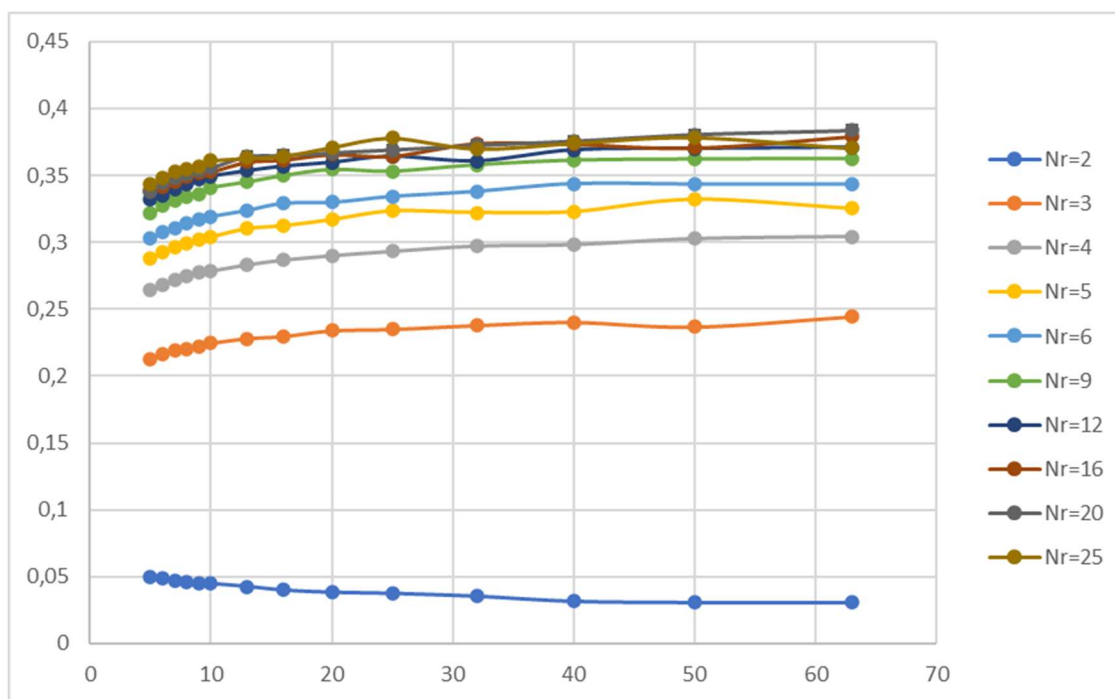


Figure 26: Mean values of "a" of Equation (5) obtained by regression for the z_r distributions, as a function of N_r (number of repetitions) and N_s (number of series).

When N_r is large, amounts of calculation necessary to get accurate determinations becomes huge, and there is no need for them because, in these situations, $a \ll N + 1 - 2a$. The slight irregularities that can be observed for large values of N_r are linked to this problem.

For large values of N_s , mean values of "a" tend to a limit depending on N_r . Figure 27 shows these limit values, as a function of N_r (number of repetitions). This figure shows that these limit values tend to an overall limit that was evaluated as equal to 0,407 when $N_r \rightarrow \infty$.

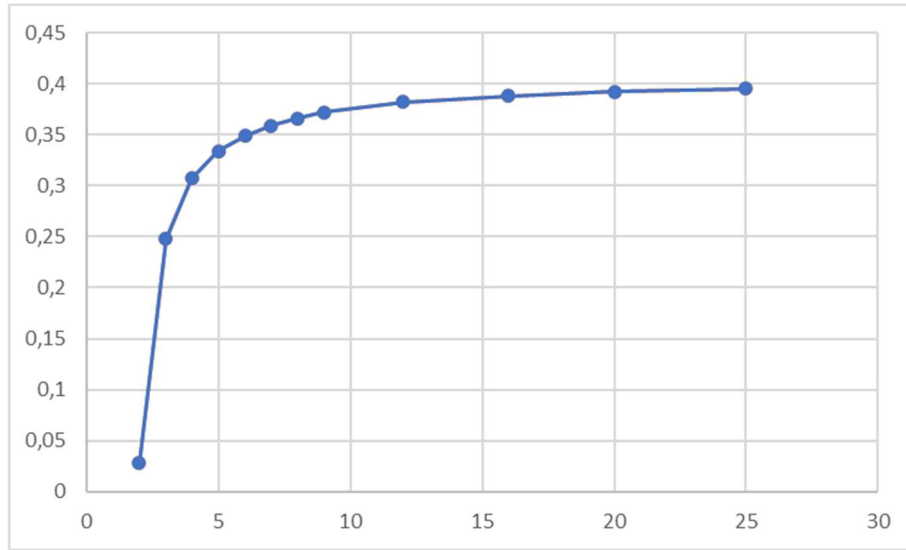


Figure 27: Limit values of “a” of Equation (5) for large values of N_s (number of series), as a function of N_r (number of repetitions).

Empirical Equation (31) provides approximations of results of Table 7 (almost all within the 2u values of Table 7). Corresponding “a” values appear of enough accuracy compared to $N + 1 - 2a$, whatever N_r and N_s.

$$a = a_{Nr} - \frac{1}{A_{Nr} \cdot N_s + B_{Nr}} \quad (31)$$

Where:

“N_r” is the number of repetitions

“N_s” is the number of series

a_{Nr} is the limit value of “a” for large values of N_r, computed as: $a_{Nr} = 0,407 - \frac{1}{3,66 \cdot N_r - 4,68}$

A_{Nr} and B_{Nr} are coefficients depending on N_r, that are computed as follows:

$A_{Nr} = -3,2$ and $B_{Nr} = -30$ when $N_r = 2$

$A_{Nr} = 2,67$ and $B_{Nr} = 15$ when $N_r = 3$

$A_{Nr} = 2,5$ and $B_{Nr} = 11$ when $N_r = 4$

$A_{Nr} = 3,35 \cdot N_r^{-0,18}$ and $B_{Nr} = 9$ when $N_r > 4$

Equation (31) can then also be written in the following way:

$$a = 0,028 + \frac{1}{3,2 \cdot N_s + 30} \text{ when } N_r = 2$$

$$a = 0,248 - \frac{1}{2,67 \cdot N_s + 15} \text{ when } N_r = 3$$

$$a = 0,307 - \frac{1}{2,5 \cdot N_s + 11} \text{ when } N_r = 4$$

$$a = 0,407 - \frac{1}{3,66 \cdot N_r - 4,68} - \frac{1}{3,35 \cdot N_r^{-0,18} \cdot N_s + 9} \text{ when } N_r > 4$$

In addition to mean values, this method provides standard deviations of “a”. This is interesting because it gives an information about the scatter with which “a” can vary around its median value. Table 7 and Figure 28 displays the results of it.

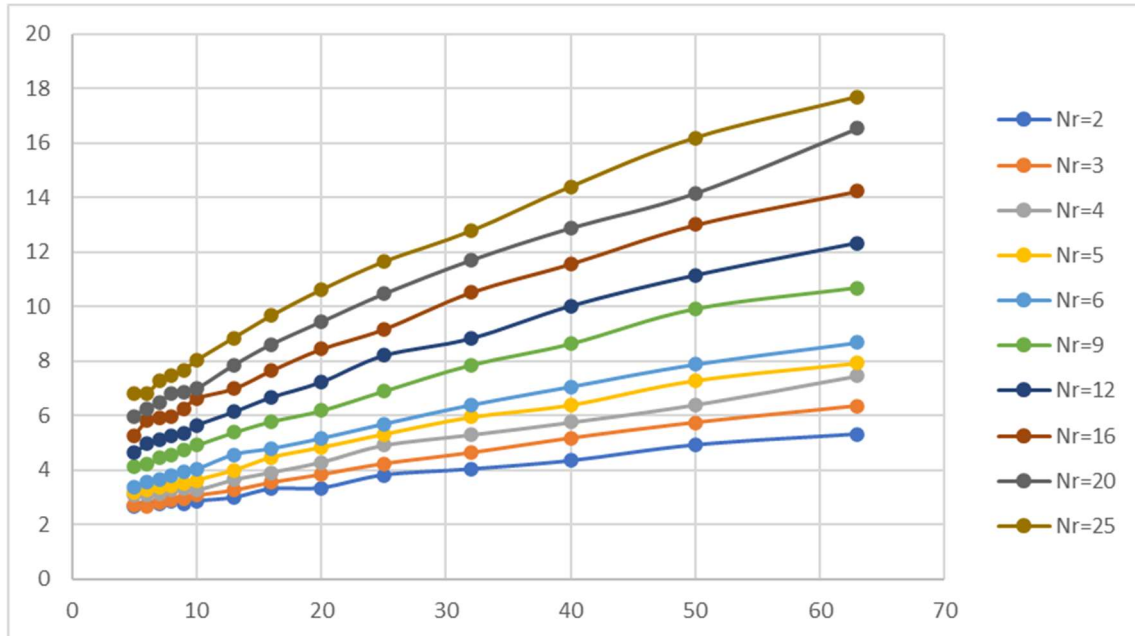


Figure 28: Standard deviations of “a” of Equation (5) obtained by regression for the zr_i distribution, as a function of N_r (number of repetitions) and N_s (number of series).

Equation (32) appeared to provide a correct approximation of standard deviations of “a” values as a function of N_r and N_s .

$$\sigma_a = \sqrt{(0,19 \cdot N_r + 0,4) \cdot N_s + (0,6 \cdot N_r + 2)} \quad (32)$$

Where:

“ N_r ” is the number of repetitions

“ N_s ” is the number of series.

In all cases, $\sigma_a > 2,5$, which means that the width of IC on “a” is always more than ± 5 , what is huge compared to differences between recommended values for it (see § 2).

5.3.4 Comparison of methods to determine abscissa corresponding to the ordinate $zr_i=1$ of the standard deviation plotting curves (determination of a standard deviation)

5.3.4.1 Quadratic mean value, Mean value or Median

The standard deviation “s” was computed by the Monte-Carlo method, using different ways of determining P_i (see § 5.3.1) and using the ZR_i determined by the quadratic mean value (QM), the mean value and the median values and the standard deviations of the distributions of zr_i , in order to determine which one is the best estimator of “s”. These values were computed to the nearest 0,01.

The corresponding results showed that:

- ✚ Whatever N_s and N_r , the QM ZR_i values, Mean ZR_i values and Median ZR_i values do not show any bias for the estimation of “s”, only if QM, mean or median is respectively used in the calculations;
- ✚ But they show significant differences between them when N_s and N_r are low;

✚ These 3 ways converge to no bias when N_s and N_r increase to large values (typically when $N_s \cdot (N_r - 1) > 40$).

In the same way, no significant difference could be seen between the standard deviations of the zr_i distributions, whatever the way to compute the ZR_i .

We then decided to adopt the quadratic mean value, as this value is well known to cause no bias when a standard deviation is determined from several estimates of it.

5.3.4.2 Computation of s_r

All methods appeared to produce significant biases when N_s and or N_r are low, even when Equation (31) is used to determine the ZR-rankits.

As shown on Figure 29 this is because the regression line produces $P_{(N+1)/2}$ that are significantly higher than the expected 0,5 value (in the case of Figure 29 for which $N_r = 2$ and $N_s = 5$, $ZR_3 = 0,568$). Consequently Equation (5), which supposes that $P_{(N+1)/2} = 0,5$ does not work properly, and its use produces biased estimations of s_r . This comes from the asymmetry of Equation (7).

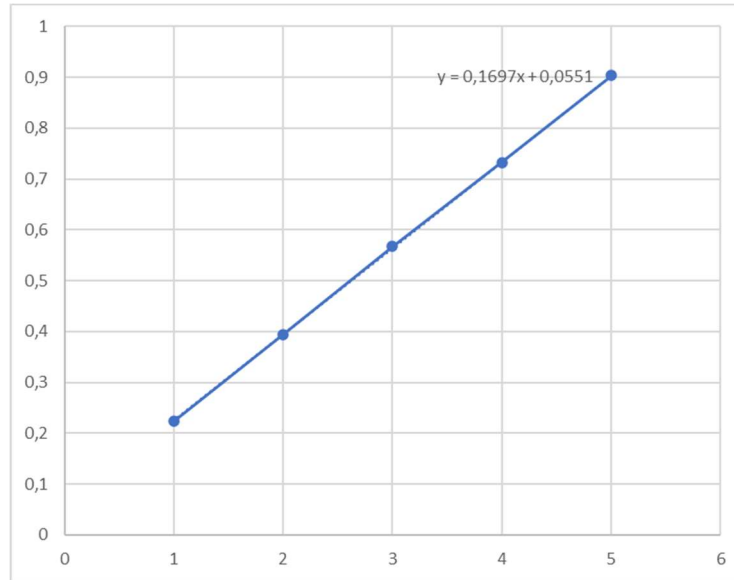


Figure 29: P_i distribution of SD probability plots for $N_r = 2$ (number of repetitions) and $N_s = 5$ (number of series).

To solve this problem, we modified Equation (5) in Equation (33) as follows:

$$P_i = \frac{i - a_1}{N + a_2} \quad (33)$$

where " P_i " is the theoretical cumulated probability of the data of rank " i "

" i " is the rank of the data,

" a_1 " and " a_2 " are constants determined as follows:

$a_1 = B_1 \cdot \log(N_s) + B_2$ with B_1 and B_2 chosen in Table 8

$a_2 = 1 - 2 \cdot a$ with " a " coming from Equation (31).

Table 8. Values of B_1 and B_2 to be used to determine a_1 and a_2 of Equation (33)

N_r	2	3	4	5	6	7	8	9 to 11	≥ 12
B_1	-0,1	0,04	0,08	0,1	0,09	0,07	0,06	0,05	$a_1 = a$ of Equation (31)
B_2	-0,255	-0,01	0,034	0,06	0,97	0,13	0,14	0,175	

Using regression straight lines using Equation (33) for determining the ZR_i produces estimations without any bias of a standard deviation from SD probability plots. The estimate of s_r corresponds to the ordinate of the abscissa "1". For example in Figure 6, the regression equation is $y = 0,1312.x + 0,6161$, $s_r = (1-0,6161)/0,1312 = 2,9$.

Note that if the regression is computed is the opposite direction of the usually plotted one (i.e. ZR_i in abscissa and data in ordinate), the estimate s_r is computed by the sum of the coefficient of regression.

Scatter linked to the determination of these standard deviations

We compared the scatter of the estimates of a standard deviation determined by Equation (33) and by the usual Equation $s = \sqrt{\sum s_i^2 / N}$. No significant differences appeared, whatever N_r and N_s (see Table A5 in annex).

5.3.5 Intervals of confidence on zr_i

The validity of the statements of § 3.5 has been verified by comparing results obtained with the method of § 3.5 and the results obtained by the Monte-Carlo method (see § 5.3.2). This check has been performed for the 2,5% centile (lower limit of the interval of confidence of 95%).

Figure 30 shows the differences in the results between these two ways of calculation.

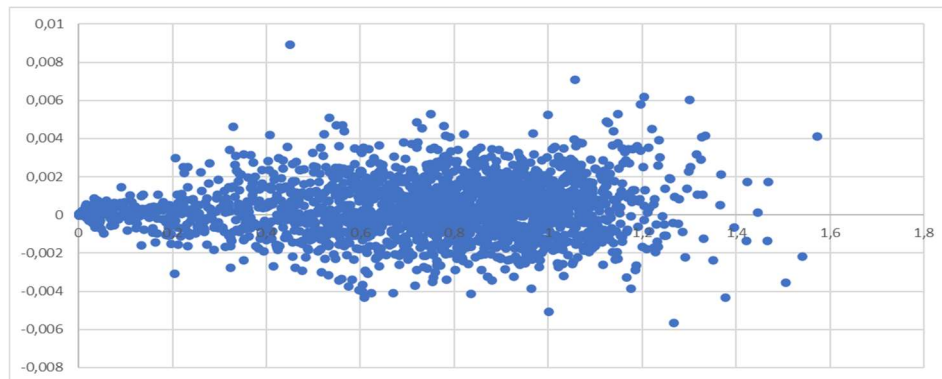


Figure 30. Differences between the centile 2,5% computed with B-rankits and with ZR_i rankits, as function of the true value of the centile, for $3 \leq N_s \leq 30$, for $2 \leq N_r \leq 25$ and for $1 \leq i \leq N$

It can be seen that no significant difference can be seen between the two series of results, validating the calculation by using the B-rankits (the max differences are within the accuracy of the determinations, as stated in § 5.3.2).

Figure 31a to Figure 31f show the results of limits of the intervals of confidence of 95% for z_i , for $N=5$, $N=9$, $N=15$ and $N=28$ as function of i and as function of ZR_i . It must be noted that, for Figure 31b, Figure 31d and Figure 31f, a choice must be made among the different ways to determine the B-rankits to be used for the computation of Z_i . To do so, we have chosen to use the Equation (2), which is the most classical one. As seen before, differences between Equations (2) to (4) are low.

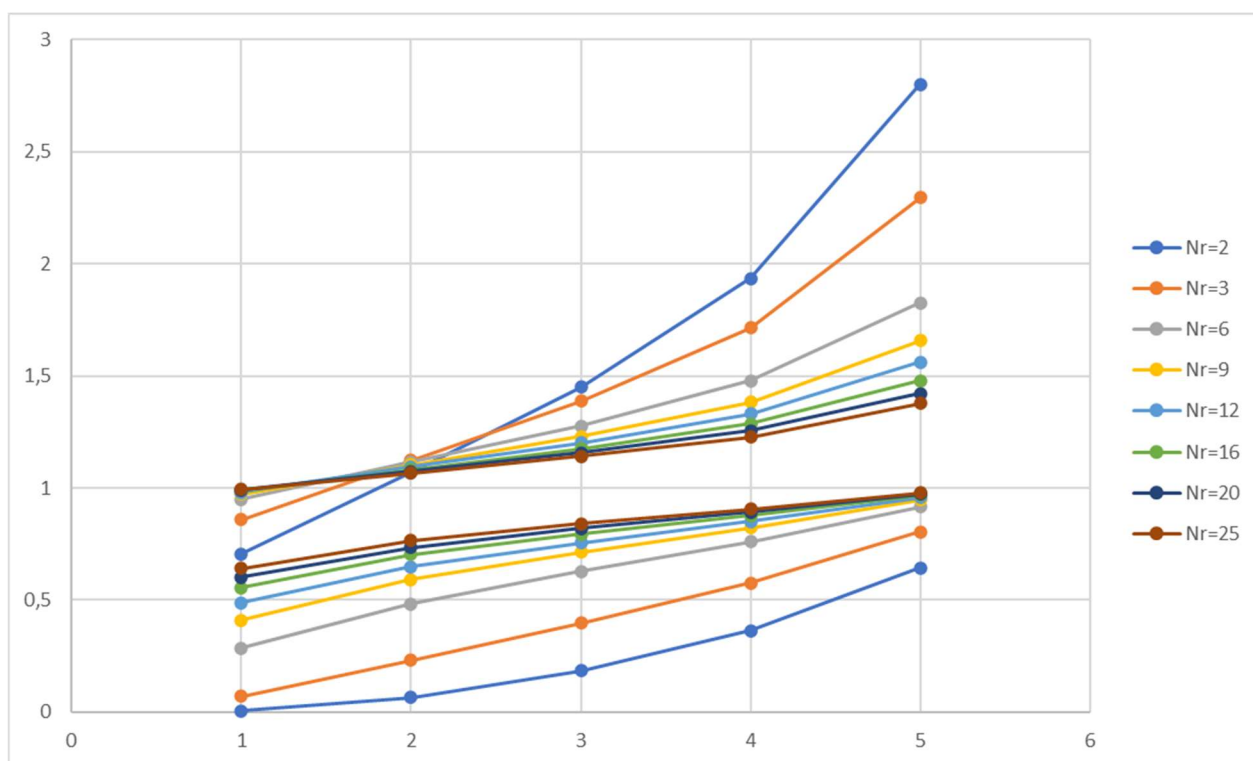


Figure 31a. Limits of intervals of confidence of 95% of s/σ , as function of i for $N_s = 5$

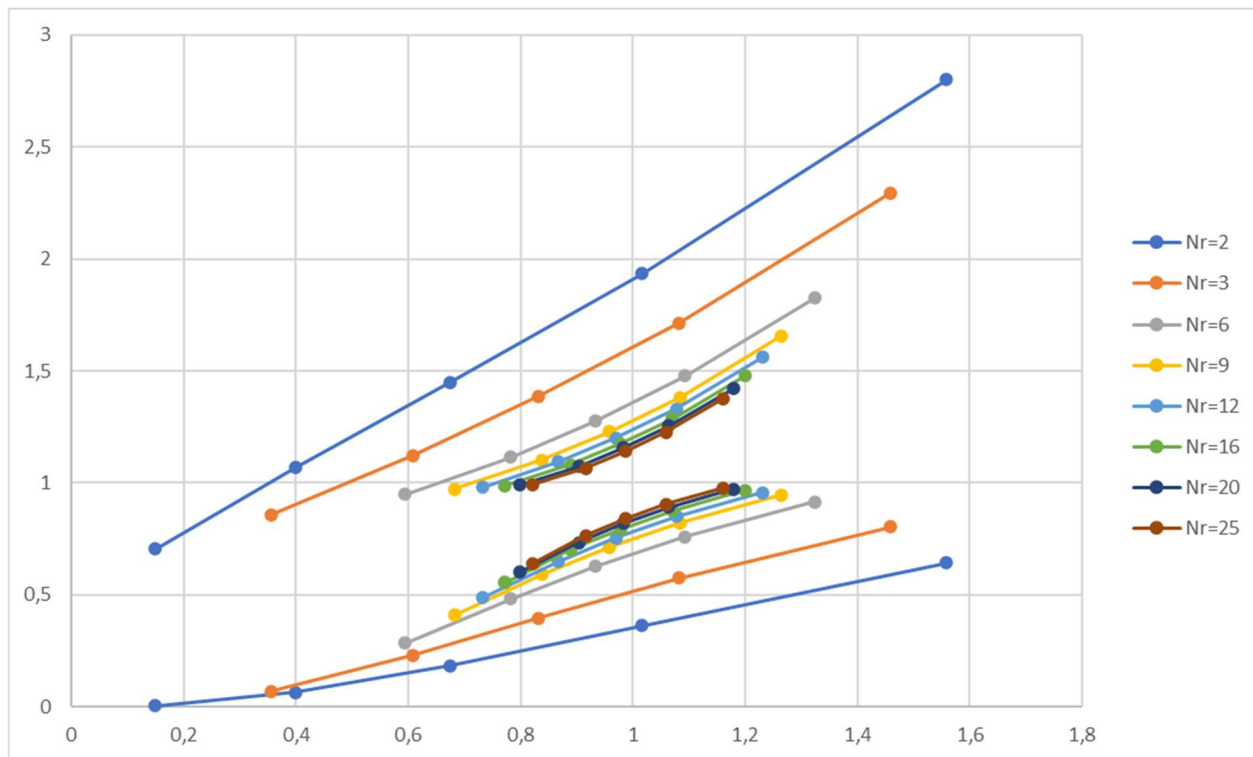


Figure 31b. Limits of intervals of confidence of 95% of s/σ , as function of ZR_i for $N_s = 5$

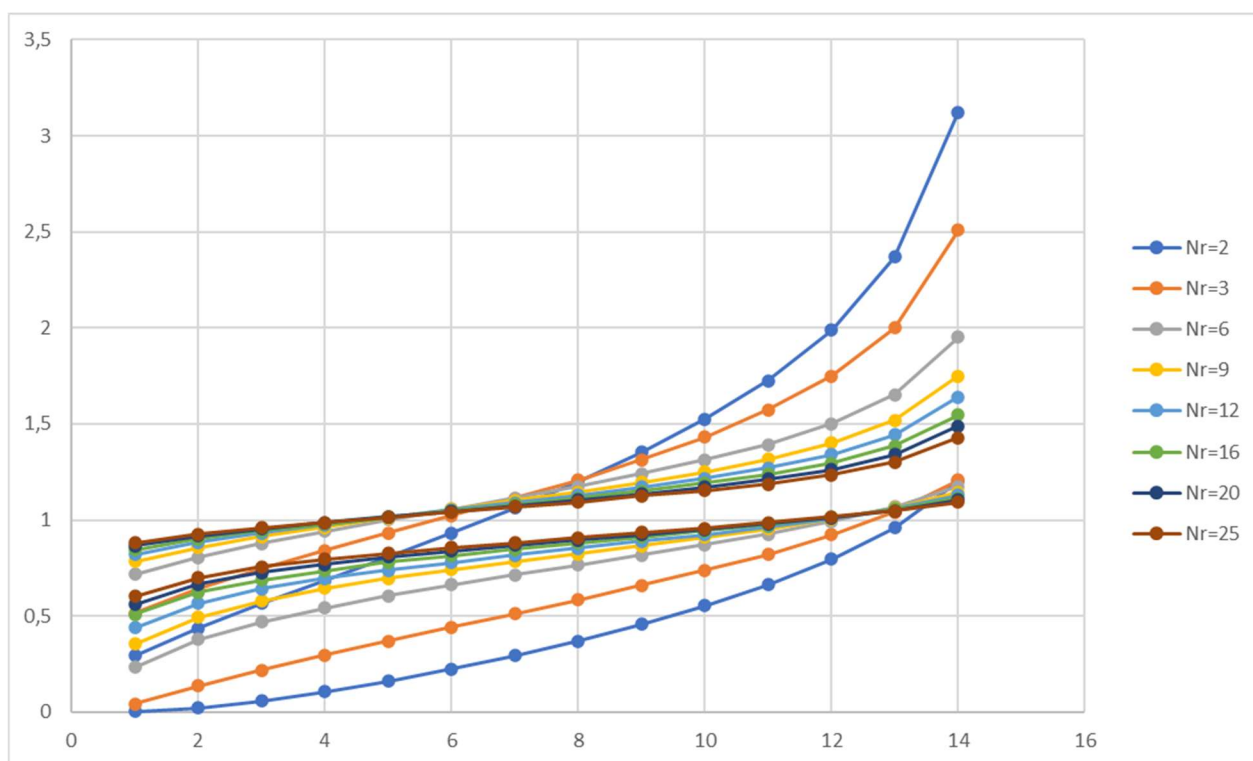


Figure 31c. Limits of intervals of confidence of 95% of s/σ , as function of i for $N_s = 14$

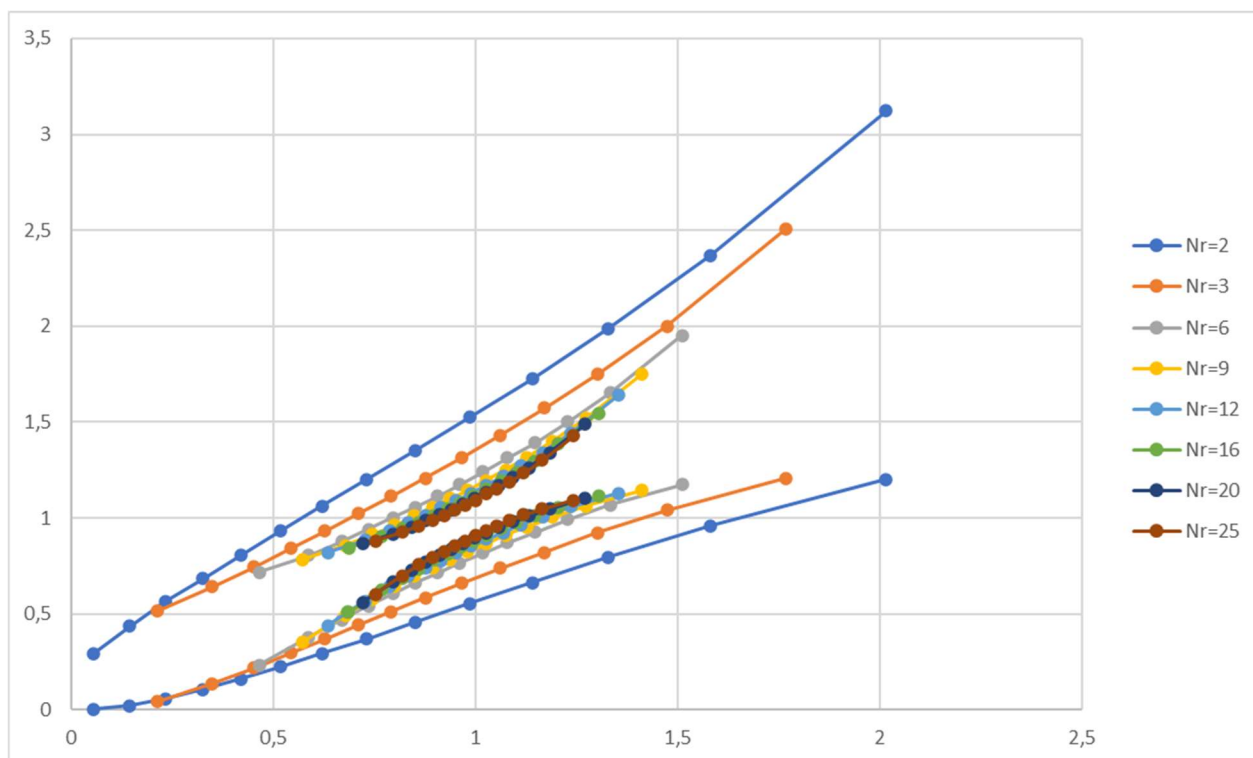


Figure 31d. Limits of intervals of confidence of 95% of s/σ , as function of ZR_i for $N_s = 14$

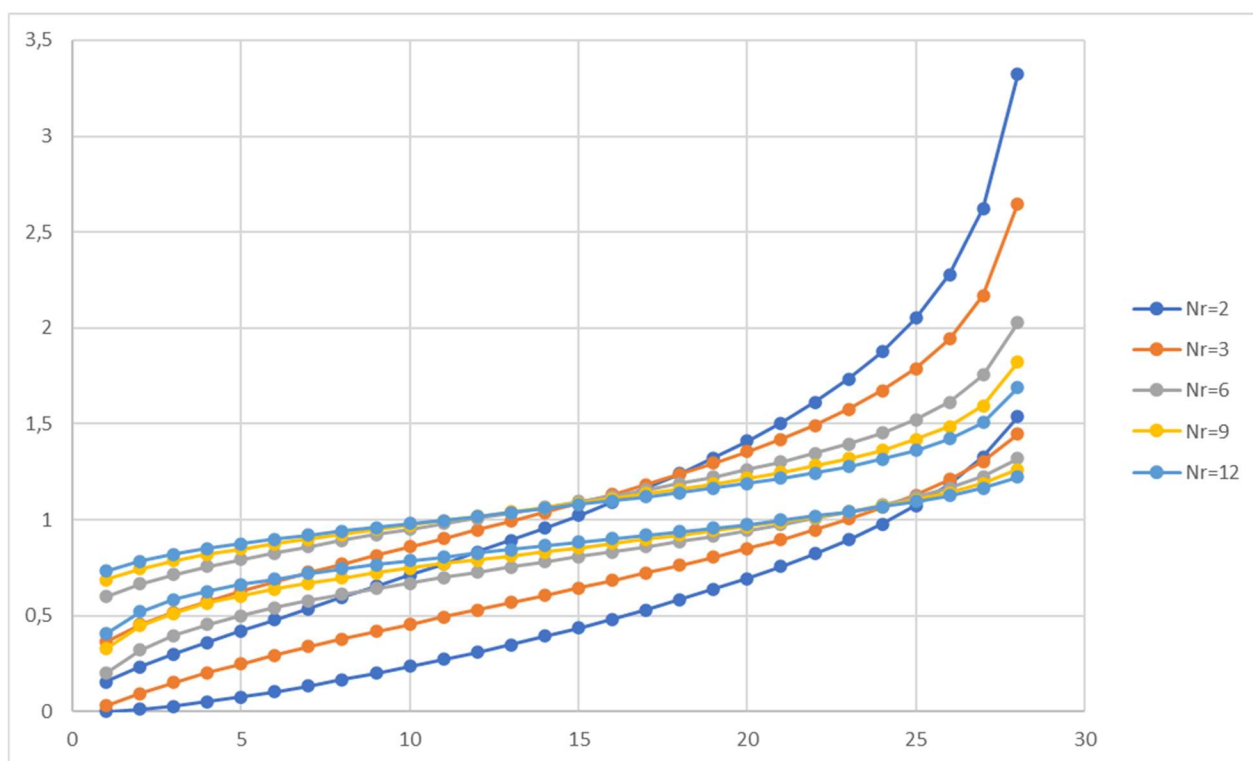


Figure 31e. Limits of intervals of confidence of 95% of s/σ , as function of i for $N_s = 28$

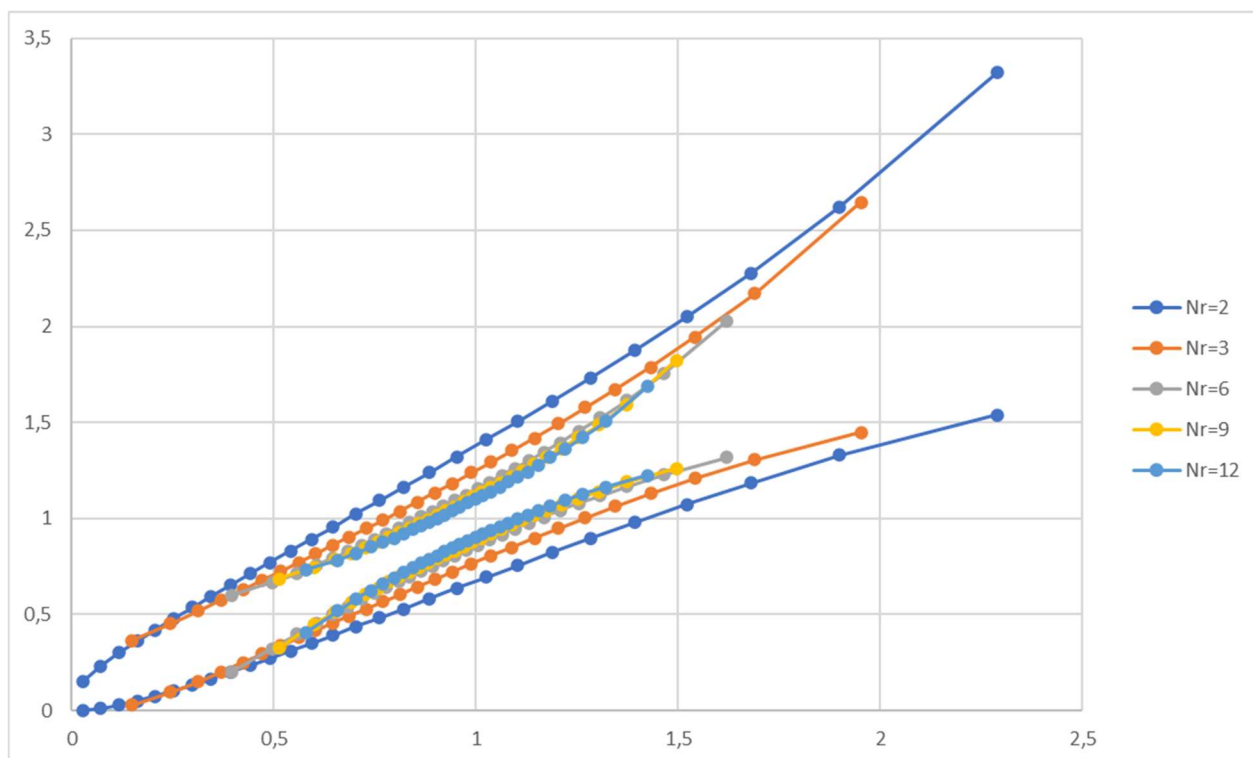


Figure 31f. Limits of intervals of confidence of 95% of s/σ , as function of ZR_i for $N_s = 28$

Note that, in practical cases, the true value of the standard deviation σ is unknown. Only its estimate “ s ” is known. However, knowledge of this value σ is necessary to plot Figure 31a to Figure 31f and use them to check whether or not a set of data falls within the corresponding confidence intervals. A future study is planned to determine techniques for overcoming this problem.

Conclusions:

Figure 31a to Figure 31f (especially Figure 31f) show that intervals of confidence tend to the Gaussian ones when N_r increases.

These figures provide much information on how the s_i scatter. However, σ is normally not known and consequently, these figures cannot be drawn in practical cases.

5.3.6 Conclusions concerning zr_i distributions for the S-distribution of a Gaussian population

No significant difference of efficiency (bias and scatter) could be seen between the different ways to compute the ZR_i (Quadratic mean ZR_i , Mean ZR_i and Median ZR_i). An empirical formula was determined to compute the most adequate values of a , as function of N_r (number of repetitions) and N_s (number of series) used to compute the standard deviation.

Confidence envelope curves of the regression line can be accurately determined with the centiles of the p_i distributions. These envelope curves tend to those of normal probability plots when N_r and N_s are large. This is consistent with well-known equations of Gaussian approximations of χ^2 distributions.

5.4 General conclusions

Cumulated probability plots can be used for checking whether a population fits a form of distribution (typically check whether a population is Gaussian, but not only), for checking whether some data are outlying and for estimating parameters of the distribution law. These three types of use do not require the same level of accuracy for the rankits needed to build up the cumulated probability plots.

Checking whether a population fits a form of distribution would ideally request envelope curves of confidence rather than straight lines, i.e. pairs of rankits. However, the habit is to use only straight lines for which a high accuracy is not necessary to check the straightness of the plot. Same comments apply to checking whether some data are outlying, as an outlying data is a data that does not pertain to the main population of them.

On the other hand, estimating parameters of the distribution requests accurate determination of the slope of the cumulated probability plot, and consequently, to use more accurate rankits than for the other uses.

Probability distributions of ordered random values from uniform distributions can be modeled by $[i-1, N]$ Binomial distributions (where i is the order of the value and N the total number of values). Mean values of these distributions can be determined with the equation $P_i = i/(N + 1)$. Approached values of the centile values (including the median values) can be determined with the binary search algorithm.

Centiles values are kept during transformations. For this reason, when no knowledge about adequate rankits is available for a certain type of distribution law, transformations of median values of the Binomial root distribution are good candidates for corresponding rankits. For the same reason, envelope confidence curves can be determined from transformations of centile values of the Binomial root distribution.

Appropriate P_i values are distributed very closely to a straight line. Usually recommended equations $((i - a)/(N + 1 - 2a))$ with $a=3/8$ or $a=0,3175$ produce rankits that are closed to the median values of the Binomial root distributions (differences between true rankits and the straight line is less than 0,01). We proposed equations to determine “ a ” as a function of “ N ”, that reduce this gap to 0,001. There is then no need to use more complicated functions of approximation (for example polynomial of higher degree) to approximate the true rankit

values. When N is large (typically more than 30), the scatter on “ a ” is so huge that no care is needed for selecting an accurate value of “ a ”.

The empirical equation $a = 0,33 - 1/(3,7.N + 18,2)$ enables to determine accurate rankits of the Binomial root distributions that can be used to determine adequate rankits for the probability plot of any form of distribution, as soon as its cumulative function is known.

Concerning normally distributed populations:

- ✚ The determination of the mean value using regression is as accurate as the usual method, whatever the way of computing the G-rankits;
- ✚ The determination of estimates of the standard deviation through the value of slope of the regression line requests to use specially determined rankits. Both median rankits and mean rankits produce biases, even for large values of N (+0,01 in most cases). However, some verifications confirmed that all modes lead to absence of bias when $N \rightarrow \infty$. The equation $a = 0,41 - 1/(1,5.N + 10)$ was empirically determined to produce rankits that avoid any bias (consequently, $a \rightarrow 0,41$ when $N \rightarrow \infty$);
- ✚ Provided that appropriate “ a ” values are used, the determination of the standard deviation is of same accuracy as the usual method. Moreover, it enables to easily detect outliers;
- ✚ The use of the centiles of the Binomial root distributions produces better confidence envelope curves of the regression line than the usual equation for regression lines.

Concerning the populations of standard deviations determined from a limited number of repetitions:

- ✚ The determination of the mean value using regression is as accurate as the usual method;
- ✚ The determination of estimates of the standard deviation through the value of slope of the regression line requests to use specially determined rankits. The usual equation $P_i = (i - a)/(N + 1 - 2.a)$ does not work properly and needs to be transformed into a equation $P_i = (i - a_1)/(N + a_2)$ where a_1 and a_2 are function of N_s (number of series) and N_r (number of repetitions). When N_s and N_r are large, $a \rightarrow 0,407$, $a_1 \rightarrow a$ and $a_2 \rightarrow 1 - 2.a$;
- ✚ Provided that appropriate “ a ” values are used, the determination of the standard deviation is of same accuracy as the usual method. Moreover, it enables to easily detect outliers;
- ✚ Confidence envelope curves of the regression line can be accurately determined with the centiles of the Binomial root distributions. These envelope curves tend to those of normal probability plots when N_r and N_s are large.

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Annex:

Detailed test results that were obtained for this study

Table A1:

Mean values, median values and centiles 0,5%, 1%, 5%, 10%, 90%, 95%, 99% and 99,5% for N = 2 to 30, i = 1 to N

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
2	1	0,33333	0,29289	0,00250	0,00501	0,02532	0,05132	0,68377	0,77639	0,90000	0,92929
2	2	0,66667	0,70711	0,07071	0,10000	0,22361	0,31623	0,94868	0,97468	0,99499	0,99750
3	1	0,25000	0,20630	0,00167	0,00334	0,01695	0,03451	0,53584	0,63160	0,78456	0,82900
3	2	0,50000	0,50000	0,04140	0,05890	0,13535	0,19580	0,80420	0,86465	0,94110	0,95860
3	3	0,75000	0,79370	0,17100	0,21544	0,36840	0,46416	0,96549	0,98305	0,99666	0,99833
4	1	0,20000	0,15910	0,00125	0,00251	0,01274	0,02600	0,43766	0,52713	0,68377	0,73409
4	2	0,40000	0,38573	0,02945	0,04200	0,09761	0,14256	0,67954	0,75140	0,85913	0,88911
4	3	0,60000	0,61427	0,11089	0,14087	0,24860	0,32046	0,85744	0,90239	0,95800	0,97055
4	4	0,80000	0,84090	0,26591	0,31623	0,47287	0,56234	0,97400	0,98726	0,99749	0,99875
5	1	0,16667	0,12945	0,00100	0,00201	0,01021	0,02085	0,36904	0,45072	0,60189	0,65343
5	2	0,33333	0,31381	0,02288	0,03268	0,07644	0,11223	0,58389	0,65741	0,77793	0,81490
5	3	0,50000	0,50000	0,08283	0,10564	0,18926	0,24664	0,75336	0,81074	0,89436	0,91717
5	4	0,66667	0,68619	0,18510	0,22207	0,34259	0,41611	0,88777	0,92356	0,96732	0,97712
5	5	0,83333	0,87055	0,34657	0,39811	0,54928	0,63096	0,97915	0,98979	0,99799	0,99900
6	1	0,14286	0,10910	0,00083	0,00167	0,00851	0,01741	0,31871	0,39304	0,53584	0,58648
6	2	0,28571	0,26445	0,01872	0,02676	0,06285	0,09260	0,51032	0,58180	0,70569	0,74601
6	3	0,42857	0,42141	0,06628	0,08473	0,15316	0,20091	0,66681	0,72866	0,82693	0,85640
6	4	0,57143	0,57859	0,14360	0,17307	0,27134	0,33319	0,79909	0,84684	0,91527	0,93372
6	5	0,71429	0,73555	0,25399	0,29431	0,41820	0,48968	0,90740	0,93715	0,97324	0,98128
6	6	0,85714	0,89090	0,41352	0,46416	0,60696	0,68129	0,98259	0,99149	0,99833	0,99917
7	1	0,12500	0,09428	0,00072	0,00143	0,00730	0,01494	0,28031	0,34816	0,48205	0,53088
7	2	0,25000	0,22849	0,01584	0,02267	0,05338	0,07882	0,45256	0,52070	0,64336	0,68491
7	3	0,37500	0,36412	0,05530	0,07080	0,12876	0,16964	0,59618	0,65874	0,76368	0,79703
7	4	0,50000	0,50000	0,11770	0,14227	0,22532	0,27860	0,72140	0,77468	0,85773	0,88230
7	5	0,62500	0,63588	0,20297	0,23632	0,34126	0,40382	0,83036	0,87124	0,92920	0,94470
7	6	0,75000	0,77151	0,31509	0,35664	0,47930	0,54744	0,92118	0,94662	0,97733	0,98416
7	7	0,87500	0,90572	0,46912	0,51795	0,65184	0,71969	0,98506	0,99270	0,99857	0,99928
8	1	0,11111	0,08300	0,00063	0,00126	0,00639	0,01308	0,25011	0,31234	0,43766	0,48433
8	2	0,22222	0,20113	0,01374	0,01966	0,04639	0,06863	0,40625	0,47068	0,58994	0,63152
8	3	0,33333	0,32052	0,04746	0,06084	0,11111	0,14685	0,53822	0,59969	0,70677	0,74217
8	4	0,44444	0,44016	0,09987	0,12095	0,19290	0,23966	0,65538	0,71076	0,80180	0,83030
8	5	0,55556	0,55984	0,16970	0,19820	0,28924	0,34462	0,76034	0,80710	0,87905	0,90013
8	6	0,66667	0,67948	0,25783	0,29323	0,40031	0,46178	0,85315	0,88889	0,93916	0,95254
8	7	0,77778	0,79887	0,36848	0,41006	0,52932	0,59375	0,93137	0,95361	0,98034	0,98626
8	8	0,88889	0,91700	0,51567	0,56234	0,68766	0,74989	0,98692	0,99361	0,99874	0,99937
9	1	0,10000	0,07412	0,00056	0,00112	0,00568	0,01164	0,22574	0,28313	0,40052	0,44495
9	2	0,20000	0,17962	0,01212	0,01736	0,04102	0,06077	0,36836	0,42914	0,54403	0,58497
9	3	0,30000	0,28624	0,04158	0,05335	0,09775	0,12950	0,49008	0,54964	0,65631	0,69261
9	4	0,40000	0,39309	0,08679	0,10526	0,16875	0,21040	0,59942	0,65506	0,74997	0,78086
9	5	0,50000	0,50000	0,14606	0,17096	0,25137	0,30097	0,69903	0,74863	0,82904	0,85394
9	6	0,60000	0,60691	0,21914	0,25003	0,34494	0,40058	0,78960	0,83125	0,89474	0,91321
9	7	0,70000	0,71376	0,30739	0,34369	0,45036	0,50992	0,87050	0,90225	0,94665	0,95842
9	8	0,80000	0,82038	0,41503	0,45597	0,57086	0,63164	0,93923	0,95898	0,98264	0,98788
9	9	0,90000	0,92588	0,55505	0,59948	0,71687	0,77426	0,98836	0,99432	0,99888	0,99944
10	1	0,09091	0,06697	0,00050	0,00100	0,00512	0,01048	0,20567	0,25887	0,36904	0,41130

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
10	2	0,18182	0,16226	0,01085	0,01554	0,03677	0,05453	0,33685	0,39416	0,50435	0,54429
10	3	0,27273	0,25857	0,03701	0,04751	0,08726	0,11583	0,44960	0,50690	0,61174	0,64820
10	4	0,36364	0,35510	0,07677	0,09321	0,15003	0,18756	0,55173	0,60662	0,70288	0,73511
10	5	0,45455	0,45169	0,12831	0,15044	0,22244	0,26732	0,64578	0,69646	0,78166	0,80908
10	6	0,54545	0,54831	0,19092	0,21834	0,30354	0,35422	0,73268	0,77756	0,84956	0,87169
10	7	0,63636	0,64490	0,26489	0,29712	0,39338	0,44827	0,81244	0,84997	0,90679	0,92323
10	8	0,72727	0,74143	0,35180	0,38826	0,49310	0,55040	0,88417	0,91274	0,95249	0,96299
10	9	0,81818	0,83774	0,45571	0,49565	0,60584	0,66315	0,94547	0,96323	0,98446	0,98915
10	10	0,90909	0,93303	0,58870	0,63096	0,74113	0,79433	0,98952	0,99488	0,99900	0,99950
11	1	0,08333	0,06107	0,00046	0,00091	0,00465	0,00953	0,18887	0,23840	0,34207	0,38225
11	2	0,16667	0,14796	0,00982	0,01407	0,03332	0,04945	0,31024	0,36436	0,46982	0,50856
11	3	0,25000	0,23579	0,03334	0,04282	0,07882	0,10477	0,41516	0,47009	0,57232	0,60850
11	4	0,33333	0,32380	0,06884	0,08366	0,13508	0,16923	0,51076	0,56437	0,66042	0,69328
11	5	0,41667	0,41189	0,11447	0,13439	0,19958	0,24053	0,59947	0,65019	0,73780	0,76680
11	6	0,50000	0,50000	0,16931	0,19398	0,27125	0,31772	0,68228	0,72875	0,80602	0,83069
11	7	0,58333	0,58811	0,23320	0,26220	0,34981	0,40053	0,75947	0,80042	0,86561	0,88553
11	8	0,66667	0,67620	0,30672	0,33958	0,43563	0,48924	0,83077	0,86492	0,91634	0,93116
11	9	0,75000	0,76421	0,39150	0,42768	0,52991	0,58484	0,89523	0,92118	0,95718	0,96666
11	10	0,83333	0,85204	0,49144	0,53018	0,63564	0,68976	0,95055	0,96668	0,98593	0,99018
11	11	0,91667	0,93893	0,61775	0,65793	0,76160	0,81113	0,99047	0,99535	0,99909	0,99954
12	1	0,07692	0,05613	0,00042	0,00084	0,00427	0,00874	0,17460	0,22092	0,31871	0,35695
12	2	0,15385	0,13598	0,00897	0,01285	0,03046	0,04524	0,28750	0,33868	0,43954	0,47703
12	3	0,23077	0,21669	0,03034	0,03898	0,07187	0,09565	0,38552	0,43811	0,53734	0,57295
12	4	0,30769	0,29758	0,06240	0,07589	0,12285	0,15419	0,47527	0,52733	0,62219	0,65522
12	5	0,38462	0,37853	0,10336	0,12147	0,18103	0,21868	0,55900	0,60914	0,69760	0,72752
12	6	0,46154	0,45951	0,15219	0,17461	0,24530	0,28817	0,63772	0,68476	0,76511	0,79147
12	7	0,53846	0,54049	0,20853	0,23489	0,31524	0,36228	0,71183	0,75470	0,82539	0,84781
12	8	0,61538	0,62147	0,27248	0,30240	0,39086	0,44100	0,78132	0,81897	0,87853	0,89664
12	9	0,69231	0,70242	0,34478	0,37781	0,47267	0,52473	0,84581	0,87715	0,92411	0,93760
12	10	0,76923	0,78331	0,42705	0,46266	0,56189	0,61448	0,90435	0,92813	0,96102	0,96966
12	11	0,84615	0,86402	0,52297	0,56046	0,66132	0,71250	0,95476	0,96954	0,98715	0,99103
12	12	0,92308	0,94387	0,64305	0,68129	0,77908	0,82540	0,99126	0,99573	0,99916	0,99958
13	1	0,07143	0,05192	0,00039	0,00077	0,00394	0,00807	0,16232	0,20582	0,29830	0,33473
13	2	0,14286	0,12579	0,00825	0,01182	0,02805	0,04169	0,26784	0,31634	0,41283	0,44902
13	3	0,21429	0,20045	0,02783	0,03578	0,06605	0,08800	0,35978	0,41010	0,50617	0,54104
13	4	0,28571	0,27528	0,05708	0,06946	0,11267	0,14161	0,44426	0,49465	0,58776	0,62064
13	5	0,35714	0,35016	0,09423	0,11083	0,16566	0,20050	0,52343	0,57262	0,66090	0,69128
13	6	0,42857	0,42508	0,13827	0,15882	0,22396	0,26373	0,59824	0,64520	0,72711	0,75457
13	7	0,50000	0,50000	0,18870	0,21288	0,28705	0,33086	0,66914	0,71295	0,78712	0,81130
13	8	0,57143	0,57492	0,24543	0,27289	0,35480	0,40176	0,73627	0,77604	0,84118	0,86173
13	9	0,64286	0,64984	0,30872	0,33910	0,42738	0,47657	0,79950	0,83434	0,88917	0,90577
13	10	0,71429	0,72472	0,37936	0,41224	0,50535	0,55574	0,85839	0,88733	0,93054	0,94292
13	11	0,78571	0,79955	0,45896	0,49383	0,58990	0,64022	0,91200	0,93395	0,96422	0,97217
13	12	0,85714	0,87421	0,55098	0,58717	0,68366	0,73216	0,95831	0,97195	0,98818	0,99175
13	13	0,92857	0,94808	0,66527	0,70170	0,79418	0,83768	0,99193	0,99606	0,99923	0,99961
14	1	0,06667	0,04831	0,00036	0,00072	0,00366	0,00750	0,15166	0,19264	0,28031	0,31508
14	2	0,13333	0,11702	0,00764	0,01095	0,02600	0,03866	0,25067	0,29673	0,38910	0,42403
14	3	0,20000	0,18647	0,02571	0,03306	0,06110	0,08148	0,33721	0,38539	0,47826	0,51231
14	4	0,26667	0,25608	0,05259	0,06403	0,10405	0,13094	0,41698	0,46566	0,55667	0,58918
14	5	0,33333	0,32575	0,08660	0,10193	0,15272	0,18513	0,49196	0,54001	0,62743	0,65794
14	6	0,40000	0,39544	0,12671	0,14568	0,20607	0,24316	0,56311	0,60959	0,69203	0,72014
14	7	0,46667	0,46515	0,17240	0,19472	0,26358	0,30455	0,63087	0,67497	0,75120	0,77657
14	8	0,53333	0,53485	0,22343	0,24880	0,32503	0,36913	0,69545	0,73642	0,80528	0,82760
14	9	0,60000	0,60456	0,27986	0,30797	0,39041	0,43689	0,75684	0,79393	0,85432	0,87329

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
14	10	0,66667	0,67425	0,34206	0,37257	0,45999	0,50804	0,81487	0,84728	0,89807	0,91340
14	11	0,73333	0,74392	0,41082	0,44333	0,53434	0,58302	0,86906	0,89595	0,93597	0,94741
14	12	0,80000	0,81353	0,48769	0,52174	0,61461	0,66279	0,91852	0,93890	0,96694	0,97429
14	13	0,86667	0,88298	0,57597	0,61090	0,70327	0,74933	0,96134	0,97400	0,98905	0,99236
14	14	0,93333	0,95169	0,68492	0,71969	0,80736	0,84834	0,99250	0,99634	0,99928	0,99964
15	1	0,06250	0,04516	0,00033	0,00067	0,00341	0,00700	0,14230	0,18104	0,26436	0,29758
15	2	0,12500	0,10940	0,00712	0,01020	0,02423	0,03604	0,23557	0,27940	0,36789	0,40159
15	3	0,18750	0,17432	0,02389	0,03072	0,05685	0,07586	0,31729	0,36344	0,45317	0,48633
15	4	0,25000	0,23939	0,04876	0,05939	0,09666	0,12177	0,39279	0,43978	0,52851	0,56053
15	5	0,31250	0,30452	0,08011	0,09436	0,14166	0,17197	0,46397	0,51075	0,59689	0,62731
15	6	0,37500	0,36967	0,11696	0,13458	0,19087	0,22559	0,53171	0,57744	0,65971	0,68816
15	7	0,43750	0,43483	0,15873	0,17946	0,24373	0,28218	0,59647	0,64043	0,71771	0,74387
15	8	0,50000	0,50000	0,20514	0,22873	0,29999	0,34152	0,65848	0,70001	0,77127	0,79486
15	9	0,56250	0,56517	0,25613	0,28229	0,35957	0,40353	0,71782	0,75627	0,82054	0,84127
15	10	0,62500	0,63033	0,31184	0,34029	0,42256	0,46829	0,77441	0,80913	0,86542	0,88304
15	11	0,68750	0,69548	0,37269	0,40311	0,48925	0,53603	0,82803	0,85834	0,90564	0,91989
15	12	0,75000	0,76061	0,43947	0,47149	0,56022	0,60721	0,87823	0,90334	0,94061	0,95124
15	13	0,81250	0,82568	0,51367	0,54683	0,63656	0,68271	0,92414	0,94315	0,96928	0,97611
15	14	0,87500	0,89060	0,59841	0,63211	0,72060	0,76443	0,96396	0,97577	0,98980	0,99288
15	15	0,93750	0,95484	0,70242	0,73564	0,81896	0,85770	0,99300	0,99659	0,99933	0,99967
16	1	0,05882	0,04240	0,00031	0,00063	0,00320	0,00656	0,13404	0,17075	0,25011	0,28190
16	2	0,11765	0,10270	0,00666	0,00954	0,02268	0,03375	0,22217	0,26396	0,34884	0,38136
16	3	0,17647	0,16365	0,02231	0,02870	0,05315	0,07097	0,29956	0,34382	0,43049	0,46276
16	4	0,23529	0,22474	0,04545	0,05538	0,09025	0,11380	0,37122	0,41657	0,50294	0,53436
16	5	0,29412	0,28589	0,07454	0,08784	0,13211	0,16056	0,43892	0,48440	0,56897	0,59913
16	6	0,35294	0,34705	0,10862	0,12506	0,17777	0,21041	0,50351	0,54835	0,62995	0,65849
16	7	0,41176	0,40823	0,14710	0,16646	0,22669	0,26292	0,56544	0,60899	0,68659	0,71323
16	8	0,47059	0,46941	0,18969	0,21172	0,27860	0,31783	0,62496	0,66663	0,73931	0,76377
16	9	0,52941	0,53059	0,23623	0,26069	0,33337	0,37504	0,68217	0,72140	0,78828	0,81031
16	10	0,58824	0,59177	0,28677	0,31341	0,39101	0,43456	0,73708	0,77331	0,83354	0,85290
16	11	0,64706	0,65295	0,34151	0,37005	0,45165	0,49649	0,78959	0,82223	0,87494	0,89138
16	12	0,70588	0,71411	0,40087	0,43103	0,51560	0,56108	0,83944	0,86789	0,91216	0,92546
16	13	0,76471	0,77526	0,46564	0,49706	0,58343	0,62878	0,88620	0,90975	0,94462	0,95455
16	14	0,82353	0,83635	0,53724	0,56951	0,65618	0,70044	0,92903	0,94685	0,97130	0,97769
16	15	0,88235	0,89730	0,61864	0,65116	0,73604	0,77783	0,96625	0,97732	0,99046	0,99334
16	16	0,94118	0,95760	0,71810	0,74989	0,82925	0,86596	0,99344	0,99680	0,99937	0,99969
17	1	0,05556	0,03995	0,00030	0,00059	0,00301	0,00618	0,12667	0,16157	0,23730	0,26777
17	2	0,11111	0,09678	0,00626	0,00897	0,02132	0,03173	0,21021	0,25012	0,33163	0,36303
17	3	0,16667	0,15422	0,02092	0,02692	0,04990	0,06667	0,28370	0,32619	0,40992	0,44129
17	4	0,22222	0,21178	0,04256	0,05188	0,08464	0,10682	0,35187	0,39564	0,47962	0,51040
17	5	0,27778	0,26940	0,06969	0,08217	0,12377	0,15058	0,41639	0,46055	0,54339	0,57318
17	6	0,33333	0,32704	0,10139	0,11681	0,16636	0,19716	0,47807	0,52192	0,60251	0,63099
17	7	0,38889	0,38469	0,13708	0,15523	0,21191	0,24614	0,53735	0,58030	0,65771	0,68459
17	8	0,44444	0,44234	0,17644	0,19711	0,26011	0,29726	0,59449	0,63599	0,70938	0,73442
17	9	0,50000	0,50000	0,21928	0,24225	0,31083	0,35039	0,64961	0,68917	0,75775	0,78072
17	10	0,55556	0,55766	0,26558	0,29062	0,36401	0,40551	0,70274	0,73989	0,80289	0,82356
17	11	0,61111	0,61531	0,31541	0,34229	0,41970	0,46265	0,75386	0,78809	0,84477	0,86292
17	12	0,66667	0,67296	0,36901	0,39749	0,47808	0,52193	0,80284	0,83364	0,88319	0,89861
17	13	0,72222	0,73060	0,42682	0,45661	0,53945	0,58361	0,84942	0,87623	0,91783	0,93031
17	14	0,77778	0,78822	0,48960	0,52038	0,60436	0,64813	0,89318	0,91536	0,94812	0,95744
17	15	0,83333	0,84578	0,55871	0,59008	0,67381	0,71630	0,93333	0,95010	0,97308	0,97908
17	16	0,88889	0,90322	0,63697	0,66837	0,74988	0,78979	0,96827	0,97868	0,99103	0,99374
17	17	0,94444	0,96005	0,73223	0,76270	0,83843	0,87333	0,99382	0,99699	0,99941	0,99970
18	1	0,05263	0,03778	0,00028	0,00056	0,00285	0,00584	0,12008	0,15332	0,22574	0,25499

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
18	2	0,10526	0,09151	0,00590	0,00846	0,02011	0,02995	0,19947	0,23766	0,31602	0,34635
18	3	0,15789	0,14581	0,01970	0,02536	0,04703	0,06286	0,26942	0,31026	0,39119	0,42167
18	4	0,21053	0,20024	0,04002	0,04880	0,07970	0,10064	0,33441	0,37668	0,45830	0,48841
18	5	0,26316	0,25471	0,06544	0,07719	0,11643	0,14177	0,39602	0,43888	0,51989	0,54924
18	6	0,31579	0,30921	0,09507	0,10959	0,15634	0,18549	0,45502	0,49783	0,57720	0,60548
18	7	0,36842	0,36371	0,12835	0,14544	0,19895	0,23139	0,51184	0,55405	0,63091	0,65786
18	8	0,42105	0,41823	0,16495	0,18441	0,24396	0,27922	0,56672	0,60784	0,68142	0,70682
18	9	0,47368	0,47274	0,20465	0,22630	0,29120	0,32885	0,61980	0,65940	0,72899	0,75261
18	10	0,52632	0,52726	0,24739	0,27101	0,34060	0,38020	0,67115	0,70880	0,77370	0,79535
18	11	0,57895	0,58177	0,29318	0,31858	0,39216	0,43328	0,72078	0,75604	0,81559	0,83505
18	12	0,63158	0,63629	0,34214	0,36909	0,44595	0,48816	0,76861	0,80105	0,85456	0,87165
18	13	0,68421	0,69079	0,39452	0,42280	0,50217	0,54498	0,81451	0,84366	0,89041	0,90493
18	14	0,73684	0,74529	0,45076	0,48011	0,56112	0,60398	0,85823	0,88357	0,92281	0,93456
18	15	0,78947	0,79976	0,51159	0,54170	0,62332	0,66559	0,89936	0,92030	0,95120	0,95998
18	16	0,84211	0,85419	0,57833	0,60881	0,68974	0,73058	0,93714	0,95297	0,97464	0,98030
18	17	0,89474	0,90849	0,65365	0,68398	0,76234	0,80053	0,97005	0,97989	0,99154	0,99410
18	18	0,94737	0,96222	0,74501	0,77426	0,84668	0,87992	0,99416	0,99715	0,99944	0,99972
19	1	0,05000	0,03582	0,00026	0,00053	0,00270	0,00553	0,11413	0,14587	0,21524	0,24335
19	2	0,10000	0,08678	0,00558	0,00800	0,01903	0,02835	0,18977	0,22637	0,30180	0,33111
19	3	0,15000	0,13827	0,01862	0,02396	0,04446	0,05946	0,25651	0,29580	0,37405	0,40368
19	4	0,20000	0,18989	0,03777	0,04606	0,07529	0,09514	0,31859	0,35943	0,43873	0,46816
19	5	0,25000	0,24154	0,06168	0,07278	0,10991	0,13394	0,37753	0,41912	0,49825	0,52711
19	6	0,30000	0,29322	0,08950	0,10321	0,14747	0,17513	0,43405	0,47580	0,55379	0,58179
19	7	0,35000	0,34491	0,12068	0,13683	0,18750	0,21832	0,48856	0,52997	0,60601	0,63291
19	8	0,40000	0,39660	0,15488	0,17327	0,22972	0,26327	0,54132	0,58194	0,65532	0,68090
19	9	0,45000	0,44830	0,19189	0,21235	0,27395	0,30983	0,59246	0,63189	0,70195	0,72601
19	10	0,50000	0,50000	0,23160	0,25395	0,32009	0,35793	0,64207	0,67991	0,74605	0,76840
19	11	0,55000	0,55170	0,27399	0,29805	0,36811	0,40754	0,69017	0,72605	0,78765	0,80811
19	12	0,60000	0,60340	0,31910	0,34468	0,41806	0,45868	0,73673	0,77028	0,82673	0,84512
19	13	0,65000	0,65509	0,36709	0,39399	0,47003	0,51144	0,78168	0,81250	0,86317	0,87932
19	14	0,70000	0,70678	0,41821	0,44621	0,52420	0,56595	0,82487	0,85253	0,89679	0,91050
19	15	0,75000	0,75846	0,47289	0,50175	0,58088	0,62247	0,86606	0,89009	0,92722	0,93832
19	16	0,80000	0,81011	0,53184	0,56127	0,64057	0,68141	0,90486	0,92471	0,95394	0,96223
19	17	0,85000	0,86173	0,59632	0,62595	0,70420	0,74349	0,94054	0,95554	0,97604	0,98138
19	18	0,90000	0,91322	0,66889	0,69820	0,77363	0,81023	0,97165	0,98097	0,99200	0,99442
19	19	0,95000	0,96418	0,75665	0,78476	0,85413	0,88587	0,99447	0,99730	0,99947	0,99974
20	1	0,04762	0,03406	0,00025	0,00050	0,00256	0,00525	0,10875	0,13911	0,20567	0,23273
20	2	0,09524	0,08251	0,00530	0,00759	0,01806	0,02691	0,18096	0,21611	0,28879	0,31714
20	3	0,14286	0,13147	0,01764	0,02271	0,04217	0,05642	0,24477	0,28262	0,35834	0,38713
20	4	0,19048	0,18055	0,03576	0,04361	0,07135	0,09021	0,30419	0,34366	0,42073	0,44947
20	5	0,23810	0,22967	0,05833	0,06885	0,10408	0,12693	0,36066	0,40103	0,47828	0,50661
20	6	0,28571	0,27880	0,08455	0,09754	0,13955	0,16587	0,41489	0,45558	0,53211	0,55976
20	7	0,33333	0,32795	0,11388	0,12918	0,17731	0,20666	0,46727	0,50782	0,58286	0,60961
20	8	0,38095	0,37711	0,14598	0,16342	0,21707	0,24906	0,51803	0,55803	0,63094	0,65657
20	9	0,42857	0,42626	0,18065	0,20005	0,25865	0,29293	0,56733	0,60642	0,67658	0,70091
20	10	0,47619	0,47542	0,21775	0,23896	0,30195	0,33817	0,61525	0,65307	0,71992	0,74277
20	11	0,52381	0,52458	0,25723	0,28008	0,34693	0,38475	0,66183	0,69805	0,76104	0,78225
20	12	0,57143	0,57374	0,29909	0,32342	0,39358	0,43267	0,70707	0,74135	0,79995	0,81935
20	13	0,61905	0,62289	0,34343	0,36906	0,44197	0,48197	0,75094	0,78293	0,83658	0,85402
20	14	0,66667	0,67205	0,39039	0,41714	0,49218	0,53273	0,79334	0,82269	0,87082	0,88612
20	15	0,71429	0,72120	0,44024	0,46789	0,54442	0,58511	0,83413	0,86045	0,90246	0,91545
20	16	0,76190	0,77033	0,49339	0,52172	0,59897	0,63934	0,87307	0,89592	0,93115	0,94167
20	17	0,80952	0,81945	0,55053	0,57927	0,65634	0,69581	0,90979	0,92865	0,95639	0,96424
20	18	0,85714	0,86853	0,61287	0,64166	0,71738	0,75523	0,94358	0,95783	0,97729	0,98236

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
20	19	0,90476	0,91749	0,68286	0,71121	0,78389	0,81904	0,97309	0,98194	0,99241	0,99470
20	20	0,95238	0,96594	0,76727	0,79433	0,86089	0,89125	0,99475	0,99744	0,99950	0,99975
21	1	0,04545	0,03247	0,00024	0,00048	0,00244	0,00500	0,10385	0,13295	0,19691	0,22299
21	2	0,09091	0,07864	0,00504	0,00722	0,01719	0,02562	0,17294	0,20673	0,27684	0,30429
21	3	0,13636	0,12531	0,01677	0,02159	0,04010	0,05367	0,23405	0,27055	0,34386	0,37185
21	4	0,18182	0,17209	0,03395	0,04142	0,06781	0,08577	0,29102	0,32921	0,40411	0,43216
21	5	0,22727	0,21890	0,05533	0,06532	0,09884	0,12062	0,34522	0,38441	0,45979	0,48758
21	6	0,27273	0,26574	0,08012	0,09246	0,13245	0,15755	0,39733	0,43698	0,51198	0,53924
21	7	0,31818	0,31258	0,10781	0,12235	0,16818	0,19619	0,44771	0,48739	0,56131	0,58783
21	8	0,36364	0,35943	0,13807	0,15464	0,20575	0,23632	0,49661	0,53594	0,60815	0,63373
21	9	0,40909	0,40629	0,17067	0,18912	0,24499	0,27779	0,54416	0,58280	0,65276	0,67723
21	10	0,45455	0,45314	0,20549	0,22567	0,28580	0,32051	0,59047	0,62810	0,69528	0,71847
21	11	0,50000	0,50000	0,24245	0,26421	0,32811	0,36443	0,63557	0,67189	0,73579	0,75755
21	12	0,54545	0,54686	0,28153	0,30472	0,37190	0,40953	0,67949	0,71420	0,77433	0,79451
21	13	0,59091	0,59371	0,32277	0,34724	0,41720	0,45584	0,72221	0,75501	0,81088	0,82933
21	14	0,63636	0,64057	0,36627	0,39185	0,46406	0,50339	0,76368	0,79425	0,84536	0,86193
21	15	0,68182	0,68742	0,41217	0,43869	0,51261	0,55229	0,80381	0,83182	0,87765	0,89219
21	16	0,72727	0,73426	0,46076	0,48802	0,56302	0,60267	0,84245	0,86755	0,90754	0,91988
21	17	0,77273	0,78110	0,51242	0,54021	0,61559	0,65478	0,87938	0,90116	0,93468	0,94467
21	18	0,81818	0,82791	0,56784	0,59589	0,67079	0,70898	0,91423	0,93219	0,95858	0,96605
21	19	0,86364	0,87469	0,62815	0,65614	0,72945	0,76595	0,94633	0,95990	0,97841	0,98323
21	20	0,90909	0,92136	0,69571	0,72316	0,79327	0,82706	0,97438	0,98281	0,99278	0,99496
21	21	0,95455	0,96753	0,77701	0,80309	0,86705	0,89615	0,99500	0,99756	0,99952	0,99976
22	1	0,04348	0,03102	0,00023	0,00046	0,00233	0,00478	0,09937	0,12731	0,18887	0,21403
22	2	0,08696	0,07512	0,00480	0,00689	0,01640	0,02444	0,16559	0,19812	0,26584	0,29243
22	3	0,13043	0,11970	0,01597	0,02057	0,03822	0,05117	0,22422	0,25947	0,33050	0,35771
22	4	0,17391	0,16439	0,03231	0,03943	0,06460	0,08175	0,27894	0,31591	0,38873	0,41611
22	5	0,21739	0,20911	0,05262	0,06214	0,09411	0,11490	0,33104	0,36909	0,44263	0,46987
22	6	0,26087	0,25384	0,07613	0,08789	0,12603	0,15002	0,38117	0,41980	0,49326	0,52010
22	7	0,30435	0,29859	0,10236	0,11621	0,15994	0,18674	0,42970	0,46849	0,54121	0,56744
22	8	0,34783	0,34335	0,13097	0,14676	0,19556	0,22483	0,47684	0,51546	0,58684	0,61229
22	9	0,39130	0,38810	0,16175	0,17934	0,23272	0,26416	0,52275	0,56087	0,63040	0,65490
22	10	0,43478	0,43286	0,19456	0,21381	0,27131	0,30463	0,56752	0,60484	0,67205	0,69544
22	11	0,47826	0,47762	0,22932	0,25008	0,31126	0,34619	0,61119	0,64746	0,71187	0,73402
22	12	0,52174	0,52238	0,26598	0,28813	0,35254	0,38881	0,65381	0,68874	0,74992	0,77068
22	13	0,56522	0,56714	0,30456	0,32795	0,39516	0,43248	0,69537	0,72869	0,78619	0,80544
22	14	0,60870	0,61190	0,34510	0,36960	0,43913	0,47725	0,73584	0,76728	0,82066	0,83825
22	15	0,65217	0,65665	0,38771	0,41316	0,48454	0,52316	0,77517	0,80444	0,85324	0,86903
22	16	0,69565	0,70141	0,43256	0,45879	0,53151	0,57030	0,81326	0,84006	0,88379	0,89764
22	17	0,73913	0,74616	0,47990	0,50674	0,58020	0,61883	0,84998	0,87397	0,91211	0,92387
22	18	0,78261	0,79089	0,53013	0,55737	0,63091	0,66896	0,88510	0,90589	0,93786	0,94738
22	19	0,82609	0,83561	0,58389	0,61127	0,68409	0,72106	0,91825	0,93540	0,96057	0,96769
22	20	0,86957	0,88030	0,64229	0,66950	0,74053	0,77578	0,94883	0,96178	0,97943	0,98403
22	21	0,91304	0,92488	0,70757	0,73416	0,80188	0,83441	0,97556	0,98360	0,99311	0,99520
22	22	0,95652	0,96898	0,78597	0,81113	0,87269	0,90063	0,99522	0,99767	0,99954	0,99977
23	1	0,04167	0,02969	0,00022	0,00044	0,00223	0,00457	0,09526	0,12212	0,18145	0,20575
23	2	0,08333	0,07191	0,00459	0,00658	0,01567	0,02337	0,15884	0,19020	0,25567	0,28144
23	3	0,12500	0,11458	0,01525	0,01964	0,03652	0,04890	0,21519	0,24925	0,31812	0,34459
23	4	0,16667	0,15734	0,03083	0,03762	0,06168	0,07808	0,26781	0,30364	0,37445	0,40118
23	5	0,20833	0,20015	0,05017	0,05925	0,08981	0,10971	0,31797	0,35493	0,42668	0,45336
23	6	0,25000	0,24297	0,07253	0,08375	0,12021	0,14318	0,36626	0,40390	0,47581	0,50221
23	7	0,29167	0,28580	0,09743	0,11066	0,15248	0,17816	0,41305	0,45097	0,52242	0,54834
23	8	0,33333	0,32863	0,12457	0,13965	0,18634	0,21442	0,45856	0,49643	0,56687	0,59213
23	9	0,37500	0,37147	0,15373	0,17053	0,22164	0,25182	0,50291	0,54046	0,60940	0,63385

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
23	10	0,41667	0,41431	0,18475	0,20314	0,25824	0,29027	0,54622	0,58316	0,65015	0,67365
23	11	0,45833	0,45716	0,21756	0,23742	0,29609	0,32971	0,58853	0,62461	0,68924	0,71165
23	12	0,50000	0,50000	0,25210	0,27329	0,33515	0,37012	0,62988	0,66485	0,72671	0,74790
23	13	0,54167	0,54284	0,28835	0,31076	0,37539	0,41147	0,67029	0,70391	0,76258	0,78244
23	14	0,58333	0,58569	0,32635	0,34985	0,41684	0,45378	0,70973	0,74176	0,79686	0,81525
23	15	0,62500	0,62853	0,36615	0,39060	0,45954	0,49709	0,74818	0,77836	0,82947	0,84627
23	16	0,66667	0,67137	0,40787	0,43313	0,50357	0,54144	0,78558	0,81366	0,86035	0,87543
23	17	0,70833	0,71420	0,45166	0,47758	0,54903	0,58695	0,82184	0,84752	0,88934	0,90257
23	18	0,75000	0,75703	0,49779	0,52419	0,59610	0,63374	0,85682	0,87979	0,91625	0,92747
23	19	0,79167	0,79985	0,54664	0,57332	0,64507	0,68203	0,89029	0,91019	0,94075	0,94983
23	20	0,83333	0,84266	0,59882	0,62555	0,69636	0,73219	0,92192	0,93832	0,96238	0,96917
23	21	0,87500	0,88542	0,65541	0,68188	0,75075	0,78481	0,95110	0,96348	0,98036	0,98475
23	22	0,91667	0,92809	0,71856	0,74433	0,80980	0,84116	0,97663	0,98433	0,99342	0,99541
23	23	0,95833	0,97031	0,79425	0,81855	0,87788	0,90474	0,99543	0,99777	0,99956	0,99978
24	1	0,04000	0,02847	0,00021	0,00042	0,00213	0,00438	0,09148	0,11735	0,17460	0,19809
24	2	0,08000	0,06895	0,00440	0,00630	0,01501	0,02238	0,15262	0,18289	0,24625	0,27125
24	3	0,12000	0,10987	0,01459	0,01879	0,03495	0,04682	0,20685	0,23980	0,30663	0,33239
24	4	0,16000	0,15088	0,02947	0,03598	0,05901	0,07474	0,25754	0,29227	0,36117	0,38726
24	5	0,20000	0,19192	0,04793	0,05662	0,08589	0,10497	0,30588	0,34181	0,41181	0,43795
24	6	0,24000	0,23299	0,06925	0,07999	0,11491	0,13694	0,35247	0,38914	0,45952	0,48546
24	7	0,28000	0,27406	0,09297	0,10562	0,14569	0,17033	0,39763	0,43469	0,50485	0,53041
24	8	0,32000	0,31513	0,11878	0,13320	0,17796	0,20493	0,44160	0,47873	0,54815	0,57317
24	9	0,36000	0,35621	0,14647	0,16255	0,21157	0,24058	0,48449	0,52142	0,58965	0,61399
24	10	0,40000	0,39729	0,17590	0,19351	0,24639	0,27721	0,52641	0,56289	0,62951	0,65302
24	11	0,44000	0,43837	0,20697	0,22599	0,28236	0,31475	0,56742	0,60321	0,66782	0,69039
24	12	0,48000	0,47946	0,23962	0,25994	0,31942	0,35317	0,60755	0,64244	0,70466	0,72617
24	13	0,52000	0,52054	0,27383	0,29534	0,35756	0,39245	0,64683	0,68058	0,74006	0,76038
24	14	0,56000	0,56163	0,30961	0,33218	0,39679	0,43258	0,68525	0,71764	0,77401	0,79303
24	15	0,60000	0,60271	0,34698	0,37049	0,43711	0,47359	0,72279	0,75361	0,80649	0,82410
24	16	0,64000	0,64379	0,38601	0,41035	0,47858	0,51551	0,75942	0,78843	0,83745	0,85353
24	17	0,68000	0,68487	0,42683	0,45185	0,52127	0,55840	0,79507	0,82204	0,86680	0,88122
24	18	0,72000	0,72594	0,46959	0,49515	0,56531	0,60237	0,82967	0,85431	0,89438	0,90703
24	19	0,76000	0,76701	0,51454	0,54048	0,61086	0,64753	0,86306	0,88509	0,92001	0,93075
24	20	0,80000	0,80808	0,56205	0,58819	0,65819	0,69412	0,89503	0,91411	0,94338	0,95207
24	21	0,84000	0,84912	0,61274	0,63883	0,70773	0,74246	0,92526	0,94099	0,96402	0,97053
24	22	0,88000	0,89013	0,66761	0,69337	0,76020	0,79315	0,95318	0,96505	0,98121	0,98541
24	23	0,92000	0,93105	0,72875	0,75375	0,81711	0,84738	0,97762	0,98499	0,99370	0,99560
24	24	0,96000	0,97153	0,80191	0,82540	0,88265	0,90852	0,99562	0,99787	0,99958	0,99979
25	1	0,03846	0,02735	0,00020	0,00040	0,00205	0,00421	0,08799	0,11293	0,16824	0,19098
25	2	0,07692	0,06623	0,00422	0,00605	0,01440	0,02148	0,14687	0,17612	0,23749	0,26176
25	3	0,11538	0,10553	0,01399	0,01802	0,03352	0,04491	0,19914	0,23104	0,29593	0,32101
25	4	0,15385	0,14492	0,02823	0,03447	0,05656	0,07166	0,24802	0,28172	0,34878	0,37426
25	5	0,19231	0,18435	0,04589	0,05422	0,08229	0,10062	0,29467	0,32961	0,39792	0,42351
25	6	0,23077	0,22379	0,06625	0,07655	0,11006	0,13123	0,33966	0,37540	0,44427	0,46976
25	7	0,26923	0,26324	0,08889	0,10102	0,13948	0,16317	0,38331	0,41952	0,48837	0,51357
25	8	0,30769	0,30270	0,11350	0,12733	0,17030	0,19624	0,42582	0,46221	0,53056	0,55532
25	9	0,34615	0,34215	0,13987	0,15529	0,20238	0,23032	0,46734	0,50364	0,57106	0,59524
25	10	0,38462	0,38161	0,16786	0,18476	0,23559	0,26529	0,50795	0,54393	0,61003	0,63349
25	11	0,42308	0,42108	0,19738	0,21563	0,26985	0,30111	0,54772	0,58316	0,64756	0,67021
25	12	0,46154	0,46054	0,22835	0,24786	0,30513	0,33774	0,58668	0,62138	0,68374	0,70545
25	13	0,50000	0,50000	0,26074	0,28141	0,34139	0,37514	0,62486	0,65861	0,71859	0,73926
25	14	0,53846	0,53946	0,29455	0,31626	0,37862	0,41332	0,66226	0,69487	0,75214	0,77165
25	15	0,57692	0,57892	0,32979	0,35244	0,41684	0,45228	0,69889	0,73015	0,78437	0,80262
25	16	0,61538	0,61839	0,36651	0,38997	0,45607	0,49205	0,73471	0,76441	0,81524	0,83214

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
25	17	0,65385	0,65785	0,40476	0,42894	0,49636	0,53266	0,76968	0,79762	0,84471	0,86013
25	18	0,69231	0,69730	0,44468	0,46944	0,53779	0,57418	0,80376	0,82970	0,87267	0,88650
25	19	0,73077	0,73676	0,48643	0,51163	0,58048	0,61669	0,83683	0,86052	0,89898	0,91111
25	20	0,76923	0,77621	0,53024	0,55573	0,62460	0,66034	0,86877	0,88994	0,92345	0,93375
25	21	0,80769	0,81565	0,57649	0,60208	0,67039	0,70533	0,89938	0,91771	0,94578	0,95411
25	22	0,84615	0,85508	0,62574	0,65122	0,71828	0,75198	0,92834	0,94344	0,96553	0,97177
25	23	0,88462	0,89447	0,67899	0,70407	0,76896	0,80086	0,95509	0,96648	0,98198	0,98601
25	24	0,92308	0,93377	0,73824	0,76251	0,82388	0,85313	0,97852	0,98560	0,99395	0,99578
25	25	0,96154	0,97265	0,80902	0,83176	0,88707	0,91201	0,99579	0,99795	0,99960	0,99980
26	1	0,03704	0,02631	0,00019	0,00039	0,00197	0,00404	0,08475	0,10883	0,16232	0,18436
26	2	0,07407	0,06372	0,00405	0,00581	0,01384	0,02064	0,14153	0,16983	0,22932	0,25291
26	3	0,11111	0,10153	0,01343	0,01730	0,03220	0,04315	0,19197	0,22289	0,28594	0,31038
26	4	0,14815	0,13942	0,02710	0,03308	0,05431	0,06883	0,23917	0,27190	0,33721	0,36209
26	5	0,18519	0,17735	0,04401	0,05201	0,07899	0,09662	0,28425	0,31824	0,38492	0,40998
26	6	0,22222	0,21529	0,06351	0,07339	0,10560	0,12597	0,32774	0,36260	0,42998	0,45500
26	7	0,25926	0,25325	0,08516	0,09680	0,13377	0,15659	0,36997	0,40535	0,47290	0,49772
26	8	0,29630	0,29120	0,10867	0,12196	0,16328	0,18827	0,41112	0,44677	0,51402	0,53848
26	9	0,33333	0,32916	0,13385	0,14866	0,19396	0,22089	0,45134	0,48700	0,55354	0,57752
26	10	0,37037	0,36713	0,16054	0,17677	0,22570	0,25436	0,49071	0,52616	0,59163	0,61500
26	11	0,40741	0,40509	0,18865	0,20620	0,25842	0,28862	0,52929	0,56434	0,62839	0,65104
26	12	0,44444	0,44305	0,21811	0,23687	0,29208	0,32361	0,56714	0,60158	0,66388	0,68571
26	13	0,48148	0,48102	0,24888	0,26876	0,32664	0,35932	0,60426	0,63791	0,69816	0,71906
26	14	0,51852	0,51898	0,28094	0,30184	0,36209	0,39574	0,64068	0,67336	0,73124	0,75112
26	15	0,55556	0,55695	0,31429	0,33612	0,39842	0,43286	0,67639	0,70792	0,76313	0,78189
26	16	0,59259	0,59491	0,34896	0,37161	0,43566	0,47071	0,71138	0,74158	0,79380	0,81135
26	17	0,62963	0,63287	0,38500	0,40837	0,47384	0,50929	0,74564	0,77430	0,82323	0,83946
26	18	0,66667	0,67084	0,42248	0,44646	0,51300	0,54866	0,77911	0,80604	0,85134	0,86615
26	19	0,70370	0,70880	0,46152	0,48598	0,55323	0,58888	0,81173	0,83672	0,87804	0,89133
26	20	0,74074	0,74675	0,50228	0,52710	0,59465	0,63003	0,84341	0,86623	0,90320	0,91484
26	21	0,77778	0,78471	0,54500	0,57002	0,63740	0,67226	0,87403	0,89440	0,92661	0,93649
26	22	0,81481	0,82265	0,59002	0,61508	0,68176	0,71575	0,90338	0,92101	0,94799	0,95599
26	23	0,85185	0,86058	0,63791	0,66279	0,72810	0,76083	0,93117	0,94569	0,96692	0,97290
26	24	0,88889	0,89847	0,68962	0,71406	0,77711	0,80803	0,95685	0,96780	0,98270	0,98657
26	25	0,92593	0,93628	0,74709	0,77068	0,83017	0,85847	0,97936	0,98616	0,99419	0,99595
26	26	0,96296	0,97369	0,81564	0,83768	0,89117	0,91525	0,99596	0,99803	0,99961	0,99981
27	1	0,03571	0,02535	0,00019	0,00037	0,00190	0,00389	0,08175	0,10502	0,15681	0,17818
27	2	0,07143	0,06139	0,00390	0,00559	0,01332	0,01987	0,13657	0,16397	0,22170	0,24464
27	3	0,10714	0,09781	0,01292	0,01664	0,03098	0,04153	0,18530	0,21530	0,27661	0,30042
27	4	0,14286	0,13432	0,02604	0,03180	0,05223	0,06622	0,23094	0,26274	0,32637	0,35067
27	5	0,17857	0,17086	0,04228	0,04997	0,07594	0,09292	0,27454	0,30763	0,37273	0,39727
27	6	0,21429	0,20742	0,06098	0,07048	0,10148	0,12112	0,31663	0,35062	0,41656	0,44112
27	7	0,25000	0,24398	0,08173	0,09293	0,12852	0,15052	0,35752	0,39210	0,45836	0,48278
27	8	0,28571	0,28055	0,10424	0,11702	0,15682	0,18093	0,39739	0,43230	0,49844	0,52258
27	9	0,32143	0,31712	0,12832	0,14257	0,18622	0,21222	0,43638	0,47139	0,53702	0,56076
27	10	0,35714	0,35370	0,15383	0,16945	0,21662	0,24431	0,47458	0,50948	0,57425	0,59747
27	11	0,39286	0,39027	0,18067	0,19756	0,24793	0,27712	0,51204	0,54664	0,61023	0,63283
27	12	0,42857	0,42685	0,20876	0,22683	0,28012	0,31063	0,54881	0,58293	0,64503	0,66691
27	13	0,46429	0,46342	0,23806	0,25723	0,31314	0,34481	0,58491	0,61839	0,67871	0,69978
27	14	0,50000	0,50000	0,26855	0,28871	0,34697	0,37963	0,62037	0,65303	0,71129	0,73145
27	15	0,53571	0,53658	0,30022	0,32129	0,38161	0,41509	0,65519	0,68686	0,74277	0,76194
27	16	0,57143	0,57315	0,33309	0,35497	0,41707	0,45119	0,68937	0,71988	0,77317	0,79124
27	17	0,60714	0,60973	0,36717	0,38977	0,45336	0,48796	0,72288	0,75207	0,80244	0,81933
27	18	0,64286	0,64630	0,40253	0,42575	0,49052	0,52542	0,75569	0,78338	0,83055	0,84617
27	19	0,67857	0,68288	0,43924	0,46298	0,52861	0,56362	0,78778	0,81378	0,85743	0,87168

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
27	20	0,71429	0,71945	0,47742	0,50156	0,56770	0,60261	0,81907	0,84318	0,88298	0,89576
27	21	0,75000	0,75602	0,51722	0,54164	0,60790	0,64248	0,84948	0,87148	0,90707	0,91827
27	22	0,78571	0,79258	0,55888	0,58344	0,64938	0,68337	0,87888	0,89852	0,92952	0,93902
27	23	0,82143	0,82914	0,60273	0,62727	0,69237	0,72546	0,90708	0,92406	0,95003	0,95772
27	24	0,85714	0,86568	0,64933	0,67363	0,73726	0,76906	0,93378	0,94777	0,96820	0,97396
27	25	0,89286	0,90219	0,69958	0,72339	0,78470	0,81470	0,95847	0,96902	0,98336	0,98708
27	26	0,92857	0,93861	0,75536	0,77830	0,83603	0,86343	0,98013	0,98668	0,99441	0,99610
27	27	0,96429	0,97465	0,82182	0,84319	0,89498	0,91825	0,99611	0,99810	0,99963	0,99981
28	1	0,03448	0,02445	0,00018	0,00036	0,00183	0,00376	0,07894	0,10147	0,15166	0,17240
28	2	0,06897	0,05922	0,00376	0,00539	0,01284	0,01916	0,13194	0,15851	0,21457	0,23688
28	3	0,10345	0,09436	0,01244	0,01603	0,02985	0,04002	0,17908	0,20820	0,26785	0,29107
28	4	0,13793	0,12958	0,02507	0,03062	0,05031	0,06379	0,22325	0,25417	0,31619	0,33994
28	5	0,17241	0,16483	0,04068	0,04809	0,07311	0,08950	0,26546	0,29769	0,36128	0,38530
28	6	0,20690	0,20010	0,05865	0,06780	0,09768	0,11663	0,30624	0,33940	0,40393	0,42804
28	7	0,24138	0,23537	0,07857	0,08935	0,12367	0,14490	0,34587	0,37967	0,44465	0,46868
28	8	0,27586	0,27065	0,10016	0,11247	0,15085	0,17413	0,38454	0,41873	0,48374	0,50756
28	9	0,31034	0,30593	0,12324	0,13697	0,17908	0,20420	0,42237	0,45673	0,52141	0,54490
28	10	0,34483	0,34122	0,14767	0,16272	0,20824	0,23502	0,45945	0,49379	0,55780	0,58085
28	11	0,37931	0,37650	0,17334	0,18963	0,23827	0,26652	0,49584	0,52998	0,59301	0,61553
28	12	0,41379	0,41178	0,20019	0,21762	0,26911	0,29867	0,53159	0,56536	0,62713	0,64901
28	13	0,44828	0,44707	0,22817	0,24666	0,30072	0,33143	0,56672	0,59996	0,66020	0,68136
28	14	0,48276	0,48236	0,25724	0,27670	0,33309	0,36479	0,60126	0,63380	0,69225	0,71260
28	15	0,51724	0,51764	0,28740	0,30775	0,36620	0,39874	0,63521	0,66691	0,72330	0,74276
28	16	0,55172	0,55293	0,31864	0,33980	0,40004	0,43328	0,66857	0,69928	0,75334	0,77183
28	17	0,58621	0,58822	0,35099	0,37287	0,43464	0,46841	0,70133	0,73089	0,78238	0,79981
28	18	0,62069	0,62350	0,38447	0,40699	0,47002	0,50416	0,73348	0,76173	0,81037	0,82666
28	19	0,65517	0,65878	0,41915	0,44220	0,50621	0,54055	0,76498	0,79176	0,83728	0,85233
28	20	0,68966	0,69407	0,45510	0,47859	0,54327	0,57763	0,79580	0,82092	0,86303	0,87676
28	21	0,72414	0,72935	0,49244	0,51626	0,58127	0,61546	0,82587	0,84915	0,88753	0,89984
28	22	0,75862	0,76463	0,53132	0,55535	0,62033	0,65413	0,85510	0,87633	0,91065	0,92143
28	23	0,79310	0,79990	0,57196	0,59607	0,66060	0,69376	0,88337	0,90232	0,93220	0,94135
28	24	0,82759	0,83517	0,61470	0,63872	0,70231	0,73454	0,91050	0,92689	0,95191	0,95932
28	25	0,86207	0,87042	0,66006	0,68381	0,74583	0,77675	0,93621	0,94969	0,96938	0,97493
28	26	0,89655	0,90564	0,70893	0,73215	0,79180	0,82092	0,95998	0,97015	0,98397	0,98756
28	27	0,93103	0,94078	0,76312	0,78543	0,84149	0,86806	0,98084	0,98716	0,99461	0,99624
28	28	0,96552	0,97555	0,82760	0,84834	0,89853	0,92106	0,99624	0,99817	0,99964	0,99982
29	1	0,03333	0,02362	0,00017	0,00035	0,00177	0,00363	0,07633	0,09815	0,14683	0,16698
29	2	0,06667	0,05720	0,00363	0,00520	0,01239	0,01849	0,12762	0,15339	0,20788	0,22960
29	3	0,10000	0,09115	0,01200	0,01546	0,02880	0,03862	0,17326	0,20156	0,25963	0,28228
29	4	0,13333	0,12517	0,02417	0,02952	0,04852	0,06154	0,21605	0,24614	0,30663	0,32984
29	5	0,16667	0,15922	0,03920	0,04635	0,07049	0,08632	0,25696	0,28837	0,35049	0,37402
29	6	0,20000	0,19328	0,05649	0,06531	0,09415	0,11246	0,29650	0,32887	0,39203	0,41570
29	7	0,23333	0,22735	0,07564	0,08604	0,11917	0,13969	0,33495	0,36800	0,43172	0,45536
29	8	0,26667	0,26143	0,09639	0,10826	0,14532	0,16783	0,37248	0,40597	0,46986	0,49334
29	9	0,30000	0,29550	0,11855	0,13179	0,17246	0,19677	0,40921	0,44294	0,50664	0,52986
29	10	0,33333	0,32958	0,14198	0,15651	0,20050	0,22642	0,44524	0,47901	0,54222	0,56507
29	11	0,36667	0,36367	0,16659	0,18231	0,22934	0,25671	0,48062	0,51427	0,57668	0,59907
29	12	0,40000	0,39775	0,19231	0,20914	0,25894	0,28760	0,51539	0,54877	0,61012	0,63196
29	13	0,43333	0,43183	0,21907	0,23693	0,28927	0,31907	0,54959	0,58254	0,64257	0,66377
29	14	0,46667	0,46592	0,24686	0,26567	0,32030	0,35109	0,58324	0,61561	0,67408	0,69456
29	15	0,50000	0,50000	0,27565	0,29534	0,35200	0,38366	0,61634	0,64800	0,70466	0,72435
29	16	0,53333	0,53408	0,30544	0,32592	0,38439	0,41676	0,64891	0,67970	0,73433	0,75314
29	17	0,56667	0,56817	0,33623	0,35743	0,41746	0,45041	0,68093	0,71073	0,76307	0,78093
29	18	0,60000	0,60225	0,36804	0,38988	0,45123	0,48461	0,71240	0,74106	0,79086	0,80769

N	i	Mean value	Median value	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
29	19	0,63333	0,63633	0,40093	0,42332	0,48573	0,51938	0,74329	0,77066	0,81769	0,83341
29	20	0,66667	0,67042	0,43493	0,45778	0,52099	0,55476	0,77358	0,79950	0,84349	0,85802
29	21	0,70000	0,70450	0,47014	0,49336	0,55706	0,59079	0,80323	0,82754	0,86821	0,88145
29	22	0,73333	0,73857	0,50666	0,53014	0,59403	0,62752	0,83217	0,85468	0,89174	0,90361
29	23	0,76667	0,77265	0,54464	0,56828	0,63200	0,66505	0,86031	0,88083	0,91396	0,92436
29	24	0,80000	0,80672	0,58430	0,60797	0,67113	0,70350	0,88754	0,90585	0,93469	0,94351
29	25	0,83333	0,84078	0,62598	0,64951	0,71163	0,74304	0,91368	0,92951	0,95365	0,96080
29	26	0,86667	0,87483	0,67016	0,69337	0,75386	0,78395	0,93846	0,95148	0,97048	0,97583
29	27	0,90000	0,90885	0,71772	0,74037	0,79844	0,82674	0,96138	0,97120	0,98454	0,98800
29	28	0,93333	0,94280	0,77040	0,79212	0,84661	0,87238	0,98151	0,98761	0,99480	0,99637
29	29	0,96667	0,97638	0,83302	0,85317	0,90185	0,92367	0,99637	0,99823	0,99965	0,99983
30	1	0,03226	0,02284	0,00017	0,00034	0,00171	0,00351	0,07388	0,09503	0,14230	0,16189
30	2	0,06452	0,05532	0,00350	0,00502	0,01198	0,01787	0,12357	0,14860	0,20159	0,22275
30	3	0,09677	0,08814	0,01159	0,01493	0,02782	0,03731	0,16781	0,19533	0,25190	0,27400
30	4	0,12903	0,12104	0,02333	0,02850	0,04685	0,05944	0,20930	0,23860	0,29762	0,32031
30	5	0,16129	0,15397	0,03782	0,04472	0,06806	0,08335	0,24899	0,27961	0,34033	0,36338
30	6	0,19355	0,18691	0,05448	0,06300	0,09087	0,10858	0,28736	0,31897	0,38080	0,40403
30	7	0,22581	0,21986	0,07292	0,08297	0,11499	0,13484	0,32469	0,35701	0,41951	0,44276
30	8	0,25806	0,25281	0,09289	0,10436	0,14018	0,16198	0,36114	0,39395	0,45673	0,47988
30	9	0,29032	0,28576	0,11420	0,12699	0,16633	0,18986	0,39684	0,42993	0,49266	0,51560
30	10	0,32258	0,31872	0,13672	0,15075	0,19331	0,21842	0,43187	0,46507	0,52744	0,55008
30	11	0,35484	0,35168	0,16036	0,17554	0,22106	0,24759	0,46628	0,49944	0,56118	0,58342
30	12	0,38710	0,38464	0,18503	0,20130	0,24953	0,27733	0,50013	0,53309	0,59395	0,61570
30	13	0,41935	0,41760	0,21069	0,22796	0,27867	0,30761	0,53343	0,56605	0,62579	0,64698
30	14	0,45161	0,45056	0,23730	0,25550	0,30846	0,33840	0,56622	0,59837	0,65675	0,67730
30	15	0,48387	0,48352	0,26485	0,28390	0,33889	0,36970	0,59851	0,63005	0,68685	0,70669
30	16	0,51613	0,51648	0,29331	0,31315	0,36995	0,40149	0,63030	0,66111	0,71610	0,73515
30	17	0,54839	0,54944	0,32270	0,34325	0,40163	0,43378	0,66160	0,69154	0,74450	0,76270
30	18	0,58065	0,58240	0,35302	0,37421	0,43395	0,46657	0,69239	0,72133	0,77204	0,78931
30	19	0,61290	0,61536	0,38430	0,40605	0,46691	0,49987	0,72267	0,75047	0,79870	0,81497
30	20	0,64516	0,64832	0,41658	0,43882	0,50056	0,53372	0,75241	0,77894	0,82446	0,83964
30	21	0,67742	0,68128	0,44992	0,47256	0,53493	0,56813	0,78158	0,80669	0,84925	0,86328
30	22	0,70968	0,71424	0,48440	0,50734	0,57007	0,60316	0,81014	0,83367	0,87301	0,88580
30	23	0,74194	0,74719	0,52012	0,54327	0,60605	0,63886	0,83802	0,85982	0,89564	0,90711
30	24	0,77419	0,78014	0,55724	0,58049	0,64299	0,67531	0,86516	0,88501	0,91703	0,92708
30	25	0,80645	0,81309	0,59597	0,61920	0,68103	0,71264	0,89142	0,90913	0,93700	0,94552
30	26	0,83871	0,84603	0,63662	0,65967	0,72039	0,75101	0,91665	0,93194	0,95528	0,96218
30	27	0,87097	0,87896	0,67969	0,70238	0,76140	0,79070	0,94056	0,95315	0,97150	0,97667
30	28	0,90323	0,91186	0,72600	0,74810	0,80467	0,83219	0,96269	0,97218	0,98507	0,98841
30	29	0,93548	0,94468	0,77725	0,79841	0,85140	0,87643	0,98213	0,98802	0,99498	0,99650
30	30	0,96774	0,97716	0,83811	0,85770	0,90497	0,92612	0,99649	0,99829	0,99966	0,99983

Table A2: Centiles 0,5%, 1%, 5%, 10%, 90%, 95%, 99% and 99,5% for a selection of i values for which $i = \frac{N+1}{2}$

(values for $N < 30$ can be found in upper tables)

N	i	0,5% centile	1% centile	5% centile	10% centile	90% centile	95% centile	99% centile	99,5% centile
33	17	0,288	0,307	0,361	0,391	0,609	0,639	0,693	0,712
39	20	0,303	0,321	0,371	0,399	0,601	0,629	0,679	0,697
45	23	0,316	0,333	0,38	0,406	0,594	0,62	0,667	0,684
51	26	0,326	0,342	0,387	0,411	0,589	0,613	0,658	0,674
57	29	0,335	0,35	0,393	0,416	0,584	0,607	0,65	0,665
63	32	0,343	0,357	0,398	0,42	0,58	0,602	0,643	0,657
71	36	0,351	0,365	0,404	0,425	0,575	0,596	0,635	0,649
79	40	0,359	0,372	0,409	0,429	0,572	0,591	0,628	0,641
89	45	0,366	0,379	0,414	0,433	0,567	0,586	0,621	0,634
99	50	0,373	0,385	0,418	0,436	0,564	0,582	0,615	0,627
113	57	0,381	0,392	0,423	0,44	0,56	0,577	0,608	0,619
125	63	0,387	0,397	0,427	0,443	0,557	0,573	0,603	0,613
141	71	0,393	0,403	0,431	0,446	0,554	0,569	0,597	0,607
161	81	0,4	0,409	0,436	0,45	0,55	0,564	0,591	0,6
177	89	0,404	0,413	0,439	0,452	0,548	0,562	0,587	0,596
197	99	0,409	0,418	0,442	0,455	0,546	0,558	0,582	0,591
225	113	0,415	0,423	0,445	0,457	0,543	0,555	0,577	0,585
249	125	0,419	0,427	0,448	0,46	0,541	0,552	0,573	0,581
281	141	0,424	0,431	0,451	0,462	0,538	0,549	0,569	0,576
317	159	0,428	0,435	0,454	0,464	0,536	0,546	0,565	0,572
397	199	0,436	0,442	0,459	0,468	0,532	0,541	0,558	0,564
451	226	0,44	0,445	0,461	0,47	0,53	0,539	0,555	0,56
499	250	0,443	0,448	0,463	0,471	0,529	0,537	0,552	0,557
633	317	0,449	0,454	0,467	0,475	0,525	0,533	0,546	0,551
801	401	0,455	0,459	0,471	0,477	0,523	0,529	0,541	0,545
997	499	0,459	0,463	0,474	0,48	0,52	0,526	0,537	0,541
1247	624	0,464	0,467	0,477	0,482	0,518	0,523	0,533	0,536
1603	802	0,468	0,471	0,479	0,484	0,516	0,521	0,529	0,532
2501	1251	0,474	0,477	0,484	0,487	0,513	0,516	0,523	0,526
4001	2001	0,48	0,482	0,487	0,49	0,51	0,513	0,518	0,52
5001	2501	0,482	0,484	0,488	0,491	0,509	0,512	0,516	0,518
6301	3151	0,484	0,485	0,49	0,492	0,508	0,51	0,515	0,516
8001	4001	0,486	0,487	0,491	0,493	0,507	0,509	0,513	0,514
10001	5001	0,487	0,488	0,492	0,494	0,506	0,508	0,512	0,513

It is reminded that for all $i = \frac{N+1}{2}$, the median value of the p_i distribution is equal to 0,5.

Table A3.1: Mean values of z_i for the Gaussian distribution as function of N and i , for $N \leq 30$.

Results are only provided for i up to $N/2$. For larger i values, results can be easily determined with the equation $z_i = -z_{N-i}$.

For example, for $N=6$, $i=4$, $z_4 = -z_2 = +0,202$.

N	Values of i														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	-0,845	0													
4	-1,029	-0,297													
5	-1,163	-0,495	0												
6	-1,268	-0,642	-0,202												
7	-1,352	-0,758	-0,353	0											
8	-1,423	-0,852	-0,472	-0,152											
9	-1,485	-0,932	-0,572	-0,274	0										
10	-1,539	-1,002	-0,656	-0,376	-0,123										
11	-1,587	-1,062	-0,729	-0,462	-0,225	0									
12	-1,629	-1,115	-0,793	-0,537	-0,312	-0,102									
13	-1,668	-1,164	-0,85	-0,603	-0,388	-0,19	0								
14	-1,703	-1,208	-0,901	-0,662	-0,456	-0,267	-0,088								
15	-1,736	-1,248	-0,948	-0,715	-0,515	-0,335	-0,165	0							
16	-1,766	-1,285	-0,99	-0,763	-0,57	-0,396	-0,234	-0,077							
17	-1,794	-1,319	-1,03	-0,808	-0,62	-0,452	-0,295	-0,146	0						
18	-1,82	-1,35	-1,066	-0,848	-0,665	-0,502	-0,351	-0,208	-0,069						
19	-1,844	-1,38	-1,1	-0,886	-0,707	-0,548	-0,402	-0,264	-0,131	0					
20	-1,868	-1,408	-1,131	-0,921	-0,746	-0,591	-0,448	-0,315	-0,187	-0,062					
21	-1,889	-1,433	-1,16	-0,954	-0,781	-0,63	-0,491	-0,362	-0,238	-0,118	0				
22	-1,91	-1,458	-1,188	-0,985	-0,815	-0,667	-0,532	-0,405	-0,286	-0,17	-0,056				
23	-1,929	-1,481	-1,215	-1,014	-0,847	-0,701	-0,569	-0,446	-0,33	-0,218	-0,108	0			
24	-1,947	-1,503	-1,239	-1,041	-0,877	-0,734	-0,604	-0,484	-0,37	-0,262	-0,156	-0,052			
25	-1,966	-1,524	-1,263	-1,067	-0,905	-0,764	-0,637	-0,519	-0,409	-0,303	-0,2	-0,1	0		
26	-1,982	-1,544	-1,285	-1,091	-0,932	-0,793	-0,668	-0,553	-0,444	-0,341	-0,241	-0,144	-0,048		
27	-1,998	-1,563	-1,307	-1,115	-0,957	-0,82	-0,697	-0,584	-0,478	-0,377	-0,28	-0,185	-0,092	0	
28	-2,014	-1,582	-1,327	-1,137	-0,981	-0,846	-0,725	-0,614	-0,51	-0,411	-0,316	-0,224	-0,134	-0,044	
29	-2,029	-1,599	-1,346	-1,158	-1,004	-0,871	-0,751	-0,642	-0,54	-0,443	-0,35	-0,26	-0,172	-0,086	0
30	-2,043	-1,615	-1,365	-1,178	-1,026	-0,894	-0,777	-0,669	-0,568	-0,473	-0,382	-0,295	-0,209	-0,125	-0,041

Table A3.2:
Standard deviations of z_i for the Gaussian distribution,
as function of N and option used to determine P_i , for $N \leq 30$.

N	“a” value used to determine P_i (see Equation (5))							
	0,5	3/8	0,3175*	see**	see***	0	Median value of p_i	p_i corresponding to Z_i
3	0,458	0,510	0,541	0,541	0,541	0,658	0,542	0,525
4	0,381	0,417	0,437	0,438	0,437	0,517	0,437	0,424
5	0,333	0,360	0,376	0,376	0,376	0,436	0,375	0,365
6	0,301	0,322	0,334	0,335	0,334	0,382	0,334	0,325
7	0,276	0,294	0,304	0,304	0,304	0,343	0,304	0,296
8	0,257	0,272	0,280	0,281	0,280	0,314	0,280	0,274
9	0,241	0,254	0,261	0,262	0,261	0,290	0,261	0,255
10	0,228	0,239	0,245	0,246	0,245	0,271	0,245	0,240
11	0,217	0,227	0,232	0,233	0,232	0,255	0,232	0,227
12	0,207	0,216	0,221	0,221	0,221	0,242	0,221	0,217
13	0,199	0,207	0,212	0,212	0,211	0,230	0,212	0,208
14	0,191	0,199	0,203	0,203	0,203	0,220	0,203	0,199
15	0,184	0,192	0,195	0,195	0,195	0,211	0,195	0,192
16	0,178	0,185	0,188	0,188	0,188	0,203	0,188	0,185
17	0,172	0,179	0,182	0,182	0,182	0,195	0,182	0,179
18	0,168	0,174	0,177	0,177	0,176	0,189	0,176	0,174
19	0,163	0,168	0,171	0,171	0,171	0,183	0,171	0,168
20	0,159	0,164	0,166	0,167	0,166	0,177	0,166	0,164
21	0,155	0,160	0,162	0,162	0,162	0,172	0,162	0,160
22	0,152	0,156	0,158	0,158	0,158	0,168	0,158	0,156
23	0,148	0,153	0,155	0,155	0,154	0,164	0,155	0,152
24	0,145	0,149	0,151	0,151	0,151	0,160	0,151	0,149
25	0,142	0,146	0,147	0,148	0,147	0,156	0,147	0,145
26	0,139	0,143	0,145	0,145	0,145	0,153	0,145	0,143
27	0,137	0,140	0,142	0,142	0,142	0,150	0,142	0,140
28	0,134	0,137	0,139	0,139	0,139	0,146	0,139	0,137
29	0,132	0,135	0,136	0,137	0,136	0,143	0,136	0,135
30	0,130	0,133	0,134	0,134	0,134	0,141	0,134	0,132

$$*: P_N = 0,5^{\frac{1}{N}}, P_1 = 1 - P_N. \quad **: 0,5 \cdot \left(N + 1 - \frac{N-1}{2 \times 0,5^{\frac{1}{N}} - 1} \right) \quad ***: 0,5 \cdot \left(N + 1 - \frac{N-1}{\left(2 \times 0,5^{\frac{1}{N}} - 1 \right) \left(1 + \frac{1}{20N+100} \right)} \right)$$

Table A4: Values of zr_i for S-distribution, as function of N_s , N_r and i for $N_s \leq 30$.

N_s : Number of series, N_r : Number of repetitions,

QM: quadratic mean value, Mean: mean value, Med: Median, C2,5%: 2,5% centile, C97,5%: 97,5% centile.

N_s	N_r	i	QM	Mean	Med	C2,5%	C97,5%
5	2	1	0,287	0,216	0,163	0,007	0,705
5	2	2	0,52	0,447	0,404	0,066	1,068
5	2	3	0,785	0,712	0,676	0,185	1,45
5	2	4	1,12	1,044	1,006	0,363	1,935
5	2	5	1,664	1,569	1,515	0,642	2,801
5	3	1	0,447	0,396	0,372	0,071	0,858
5	3	2	0,67	0,63	0,613	0,231	1,123
5	3	3	0,885	0,848	0,832	0,398	1,389
5	3	4	1,133	1,095	1,076	0,576	1,715
5	3	5	1,51	1,461	1,43	0,804	2,295
5	6	1	0,632	0,609	0,607	0,286	0,949
5	6	2	0,806	0,789	0,785	0,484	1,116
5	6	3	0,953	0,938	0,934	0,627	1,276
5	6	4	1,112	1,097	1,089	0,761	1,479
5	6	5	1,343	1,323	1,306	0,917	1,826
5	9	1	0,708	0,693	0,694	0,41	0,973
5	9	2	0,853	0,843	0,841	0,591	1,103
5	9	3	0,971	0,962	0,959	0,713	1,229
5	9	4	1,097	1,087	1,082	0,822	1,383
5	9	5	1,277	1,264	1,251	0,946	1,657
5	12	1	0,75	0,739	0,741	0,487	0,981
5	12	2	0,876	0,869	0,869	0,649	1,094
5	12	3	0,978	0,972	0,969	0,755	1,201
5	12	4	1,086	1,079	1,074	0,851	1,332
5	12	5	1,239	1,229	1,218	0,958	1,561
5	16	1	0,786	0,778	0,781	0,554	0,989
5	16	2	0,897	0,891	0,891	0,702	1,083
5	16	3	0,985	0,98	0,978	0,794	1,174
5	16	4	1,076	1,071	1,067	0,876	1,286
5	16	5	1,205	1,198	1,189	0,968	1,478
5	20	1	0,809	0,803	0,806	0,603	0,991
5	20	2	0,909	0,905	0,905	0,733	1,075
5	20	3	0,988	0,984	0,982	0,82	1,156
5	20	4	1,068	1,064	1,061	0,893	1,254
5	20	5	1,183	1,177	1,171	0,973	1,421
5	25	1	0,83	0,826	0,828	0,642	0,993
5	25	2	0,921	0,917	0,917	0,764	1,067
5	25	3	0,99	0,987	0,987	0,842	1,14
5	25	4	1,062	1,059	1,057	0,904	1,226
5	25	5	1,164	1,159	1,153	0,977	1,376
6	2	1	0,247	0,183	0,137	0,005	0,613

N_s	N_r	i	QM	Mean	Med	C2,5%	C97,5%
6	2	2	0,442	0,378	0,339	0,054	0,921
6	2	3	0,653	0,59	0,556	0,149	1,221
6	2	4	0,899	0,836	0,806	0,283	1,561
6	2	5	1,218	1,15	1,118	0,466	2,021
6	2	6	1,742	1,655	1,604	0,741	2,864
6	3	1	0,408	0,362	0,34	0,066	0,782
6	3	2	0,606	0,569	0,554	0,21	1,014
6	3	3	0,785	0,753	0,74	0,354	1,225
6	3	4	0,975	0,943	0,93	0,501	1,46
6	3	5	1,204	1,17	1,154	0,666	1,771
6	3	6	1,566	1,521	1,489	0,883	2,339
6	6	1	0,604	0,582	0,58	0,278	0,9
6	6	2	0,762	0,747	0,744	0,464	1,047
6	6	3	0,888	0,875	0,871	0,594	1,18
6	6	4	1,013	1,001	0,996	0,709	1,325
6	6	5	1,157	1,144	1,136	0,823	1,512
6	6	6	1,376	1,358	1,339	0,963	1,855
6	9	1	0,684	0,67	0,672	0,401	0,933
6	9	2	0,818	0,808	0,808	0,572	1,049
6	9	3	0,92	0,912	0,911	0,683	1,154
6	9	4	1,02	1,012	1,009	0,775	1,267
6	9	5	1,133	1,125	1,119	0,871	1,412
6	9	6	1,304	1,292	1,279	0,985	1,673
6	12	1	0,73	0,72	0,722	0,478	0,946
6	12	2	0,847	0,84	0,84	0,634	1,049
6	12	3	0,935	0,929	0,928	0,73	1,136
6	12	4	1,02	1,015	1,013	0,813	1,229
6	12	5	1,118	1,111	1,106	0,894	1,354
6	12	6	1,262	1,253	1,242	0,992	1,573
6	16	1	0,767	0,759	0,762	0,545	0,958
6	16	2	0,87	0,866	0,866	0,685	1,043
6	16	3	0,947	0,943	0,943	0,77	1,118
6	16	4	1,021	1,017	1,016	0,842	1,202
6	16	5	1,103	1,099	1,095	0,915	1,305
6	16	6	1,225	1,219	1,21	0,997	1,49
6	20	1	0,793	0,787	0,791	0,591	0,965
6	20	2	0,886	0,882	0,883	0,721	1,04
6	20	3	0,955	0,952	0,951	0,798	1,109
6	20	4	1,02	1,017	1,015	0,865	1,18
6	20	5	1,093	1,089	1,085	0,927	1,273
6	20	6	1,201	1,196	1,188	0,999	1,437

N_s	N_r	i	QM	Mean	Med	C2,5%	C97,5%
6	25	1	0,815	0,811	0,814	0,634	0,969
6	25	2	0,899	0,896	0,896	0,753	1,038
6	25	3	0,96	0,957	0,957	0,821	1,096
6	25	4	1,017	1,015	1,014	0,879	1,156
6	25	5	1,082	1,079	1,077	0,936	1,237
6	25	6	1,178	1,174	1,168	1,003	1,384
7	2	1	0,216	0,16	0,118	0,005	0,538
7	2	2	0,383	0,327	0,291	0,046	0,804
7	2	3	0,561	0,506	0,475	0,124	1,056
7	2	4	0,758	0,703	0,676	0,234	1,328
7	2	5	0,992	0,936	0,909	0,375	1,652
7	2	6	1,298	1,236	1,205	0,558	2,092
7	2	7	1,806	1,725	1,673	0,823	2,911
7	3	1	0,377	0,334	0,314	0,061	0,727
7	3	2	0,556	0,523	0,508	0,193	0,931
7	3	3	0,714	0,684	0,673	0,32	1,114
7	3	4	0,871	0,843	0,832	0,451	1,301
7	3	5	1,045	1,017	1,005	0,585	1,518
7	3	6	1,262	1,231	1,215	0,739	1,815
7	3	7	1,61	1,568	1,536	0,944	2,37
7	6	1	0,58	0,56	0,559	0,267	0,86
7	6	2	0,727	0,714	0,711	0,446	0,996
7	6	3	0,841	0,829	0,826	0,566	1,11
7	6	4	0,947	0,936	0,933	0,668	1,227
7	6	5	1,059	1,048	1,043	0,767	1,362
7	6	6	1,195	1,183	1,174	0,87	1,542
7	6	7	1,406	1,389	1,371	1,005	1,873
7	9	1	0,664	0,651	0,652	0,394	0,901
7	9	2	0,789	0,78	0,78	0,556	1,008
7	9	3	0,882	0,874	0,872	0,66	1,1
7	9	4	0,967	0,96	0,958	0,745	1,19
7	9	5	1,056	1,049	1,046	0,826	1,296
7	9	6	1,162	1,154	1,148	0,909	1,434
7	9	7	1,326	1,315	1,301	1,014	1,687
7	12	1	0,713	0,704	0,706	0,469	0,921
7	12	2	0,823	0,817	0,816	0,62	1,011
7	12	3	0,903	0,898	0,898	0,712	1,089
7	12	4	0,977	0,972	0,97	0,786	1,169
7	12	5	1,053	1,048	1,046	0,854	1,256
7	12	6	1,142	1,137	1,132	0,926	1,37
7	12	7	1,281	1,273	1,261	1,019	1,588

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
7	16	1	0,753	0,746	0,749	0,538	0,938
7	16	2	0,85	0,845	0,846	0,674	1,015
7	16	3	0,919	0,915	0,914	0,753	1,081
7	16	4	0,982	0,979	0,977	0,82	1,145
7	16	5	1,047	1,044	1,041	0,879	1,219
7	16	6	1,124	1,12	1,116	0,942	1,318
7	16	7	1,242	1,236	1,226	1,018	1,502
7	20	1	0,779	0,774	0,777	0,583	0,942
7	20	2	0,867	0,864	0,864	0,71	1,015
7	20	3	0,93	0,927	0,926	0,782	1,073
7	20	4	0,986	0,983	0,982	0,84	1,131
7	20	5	1,043	1,04	1,037	0,894	1,198
7	20	6	1,111	1,108	1,105	0,948	1,284
7	20	7	1,214	1,21	1,202	1,017	1,444
7	25	1	0,805	0,801	0,803	0,631	0,955
7	25	2	0,883	0,88	0,881	0,741	1,017
7	25	3	0,939	0,937	0,937	0,808	1,068
7	25	4	0,989	0,987	0,986	0,86	1,117
7	25	5	1,041	1,038	1,037	0,907	1,179
7	25	6	1,101	1,098	1,095	0,96	1,258
7	25	7	1,193	1,189	1,182	1,022	1,399
8	2	1	0,192	0,142	0,104	0,004	0,48
8	2	2	0,339	0,287	0,255	0,04	0,715
8	2	3	0,492	0,441	0,413	0,107	0,936
8	2	4	0,658	0,608	0,583	0,198	1,163
8	2	5	0,845	0,796	0,772	0,312	1,416
8	2	6	1,068	1,017	0,993	0,454	1,718
8	2	7	1,363	1,307	1,278	0,635	2,144
8	2	8	1,859	1,783	1,734	0,9	2,953
8	3	1	0,354	0,313	0,294	0,056	0,68
8	3	2	0,518	0,486	0,474	0,181	0,866
8	3	3	0,659	0,632	0,621	0,298	1,026
8	3	4	0,796	0,771	0,761	0,412	1,187
8	3	5	0,941	0,916	0,906	0,529	1,365
8	3	6	1,104	1,079	1,067	0,657	1,573
8	3	7	1,311	1,283	1,268	0,8	1,857
8	3	8	1,649	1,609	1,577	0,996	2,4
8	6	1	0,563	0,543	0,542	0,263	0,832
8	6	2	0,701	0,688	0,686	0,433	0,955
8	6	3	0,806	0,795	0,793	0,547	1,057
8	6	4	0,9	0,89	0,886	0,642	1,157
8	6	5	0,995	0,985	0,981	0,728	1,266
8	6	6	1,098	1,088	1,082	0,813	1,392
8	6	7	1,227	1,215	1,207	0,912	1,564
8	6	8	1,432	1,415	1,398	1,04	1,892

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
8	9	1	0,649	0,636	0,638	0,382	0,878
8	9	2	0,767	0,759	0,759	0,542	0,973
8	9	3	0,852	0,846	0,845	0,641	1,054
8	9	4	0,929	0,923	0,921	0,719	1,132
8	9	5	1,004	0,998	0,996	0,792	1,218
8	9	6	1,086	1,08	1,076	0,862	1,317
8	9	7	1,186	1,179	1,173	0,939	1,453
8	9	8	1,344	1,334	1,319	1,04	1,705
8	12	1	0,697	0,688	0,692	0,459	0,897
8	12	2	0,802	0,796	0,797	0,606	0,985
8	12	3	0,877	0,872	0,872	0,698	1,056
8	12	4	0,943	0,939	0,937	0,766	1,12
8	12	5	1,008	1,003	1,001	0,828	1,191
8	12	6	1,078	1,073	1,07	0,891	1,273
8	12	7	1,163	1,158	1,152	0,957	1,386
8	12	8	1,297	1,289	1,278	1,04	1,601
8	16	1	0,741	0,735	0,739	0,531	0,917
8	16	2	0,833	0,829	0,83	0,664	0,988
8	16	3	0,898	0,894	0,894	0,743	1,05
8	16	4	0,955	0,951	0,95	0,802	1,107
8	16	5	1,01	1,007	1,005	0,855	1,167
8	16	6	1,069	1,066	1,064	0,909	1,237
8	16	7	1,142	1,138	1,135	0,965	1,334
8	16	8	1,255	1,25	1,241	1,04	1,513
8	20	1	0,769	0,764	0,767	0,581	0,931
8	20	2	0,853	0,849	0,85	0,699	0,994
8	20	3	0,911	0,908	0,908	0,769	1,044
8	20	4	0,961	0,958	0,959	0,823	1,093
8	20	5	1,01	1,008	1,007	0,873	1,15
8	20	6	1,063	1,06	1,058	0,922	1,211
8	20	7	1,128	1,125	1,121	0,972	1,298
8	20	8	1,228	1,223	1,215	1,037	1,45
8	25	1	0,794	0,79	0,793	0,619	0,939
8	25	2	0,869	0,866	0,867	0,732	0,996
8	25	3	0,921	0,919	0,919	0,797	1,043
8	25	4	0,966	0,964	0,963	0,844	1,09
8	25	5	1,01	1,008	1,008	0,888	1,135
8	25	6	1,057	1,055	1,055	0,931	1,186
8	25	7	1,114	1,111	1,108	0,978	1,261
8	25	8	1,202	1,198	1,192	1,036	1,402
9	2	1	0,173	0,127	0,093	0,004	0,435
9	2	2	0,304	0,257	0,227	0,035	0,646
9	2	3	0,44	0,394	0,367	0,094	0,842
9	2	4	0,583	0,539	0,515	0,173	1,039
9	2	5	0,741	0,696	0,675	0,269	1,246

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
9	2	6	0,92	0,876	0,854	0,385	1,486
9	2	7	1,136	1,089	1,068	0,525	1,783
9	2	8	1,422	1,369	1,342	0,703	2,197
9	2	9	1,906	1,834	1,784	0,964	2,986
9	3	1	0,333	0,295	0,277	0,052	0,639
9	3	2	0,485	0,456	0,445	0,17	0,809
9	3	3	0,615	0,59	0,58	0,282	0,956
9	3	4	0,738	0,715	0,706	0,384	1,096
9	3	5	0,863	0,841	0,832	0,488	1,245
9	3	6	0,997	0,975	0,965	0,597	1,411
9	3	7	1,153	1,13	1,118	0,714	1,61
9	3	8	1,352	1,326	1,31	0,853	1,889
9	3	9	1,681	1,644	1,613	1,044	2,415
9	6	1	0,546	0,527	0,527	0,254	0,803
9	6	2	0,677	0,665	0,663	0,42	0,916
9	6	3	0,775	0,765	0,762	0,528	1,013
9	6	4	0,86	0,851	0,848	0,616	1,103
9	6	5	0,944	0,935	0,932	0,696	1,194
9	6	6	1,03	1,022	1,019	0,774	1,293
9	6	7	1,128	1,119	1,114	0,854	1,414
9	6	8	1,252	1,241	1,232	0,947	1,588
9	6	9	1,45	1,434	1,416	1,07	1,903
9	9	1	0,635	0,623	0,626	0,376	0,856
9	9	2	0,748	0,741	0,741	0,533	0,948
9	9	3	0,829	0,823	0,822	0,629	1,022
9	9	4	0,899	0,893	0,893	0,702	1,092
9	9	5	0,966	0,961	0,959	0,768	1,164
9	9	6	1,034	1,029	1,026	0,83	1,24
9	9	7	1,111	1,106	1,102	0,897	1,336
9	9	8	1,208	1,201	1,194	0,972	1,467
9	9	9	1,361	1,351	1,337	1,067	1,712
9	12	1	0,687	0,678	0,682	0,457	0,879
9	12	2	0,787	0,781	0,783	0,596	0,959
9	12	3	0,857	0,853	0,853	0,682	1,024
9	12	4	0,918	0,914	0,913	0,748	1,084
9	12	5	0,975	0,971	0,97	0,805	1,144
9	12	6	1,034	1,031	1,029	0,86	1,214
9	12	7	1,099	1,095	1,093	0,916	1,291
9	12	8	1,18	1,176	1,17	0,979	1,399
9	12	9	1,31	1,303	1,292	1,06	1,605
9	16	1	0,731	0,724	0,729	0,525	0,903
9	16	2	0,819	0,816	0,818	0,655	0,968
9	16	3	0,881	0,878	0,878	0,729	1,023
9	16	4	0,933	0,93	0,929	0,787	1,075
9	16	5	0,982	0,979	0,978	0,836	1,128

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
9	16	6	1,032	1,029	1,028	0,885	1,182
9	16	7	1,088	1,085	1,083	0,931	1,252
9	16	8	1,157	1,154	1,15	0,986	1,344
9	16	9	1,267	1,261	1,252	1,058	1,519
9	20	1	0,759	0,754	0,758	0,571	0,911
9	20	2	0,839	0,836	0,837	0,69	0,974
9	20	3	0,894	0,892	0,892	0,757	1,022
9	20	4	0,941	0,939	0,938	0,81	1,068
9	20	5	0,985	0,983	0,982	0,857	1,116
9	20	6	1,03	1,028	1,026	0,899	1,165
9	20	7	1,079	1,077	1,074	0,941	1,222
9	20	8	1,141	1,138	1,134	0,989	1,307
9	20	9	1,236	1,232	1,224	1,052	1,458
9	25	1	0,786	0,782	0,785	0,62	0,926
9	25	2	0,858	0,856	0,856	0,727	0,981
9	25	3	0,908	0,906	0,905	0,789	1,025
9	25	4	0,95	0,948	0,947	0,835	1,065
9	25	5	0,989	0,987	0,985	0,876	1,107
9	25	6	1,029	1,027	1,026	0,913	1,149
9	25	7	1,073	1,071	1,068	0,952	1,201
9	25	8	1,128	1,125	1,123	0,995	1,274
9	25	9	1,212	1,209	1,202	1,05	1,41
10	2	1	0,157	0,115	0,084	0,003	0,398
10	2	2	0,276	0,233	0,205	0,032	0,59
10	2	3	0,397	0,355	0,33	0,084	0,765
10	2	4	0,525	0,483	0,461	0,153	0,938
10	2	5	0,662	0,621	0,6	0,237	1,121
10	2	6	0,812	0,772	0,751	0,335	1,319
10	2	7	0,985	0,944	0,923	0,452	1,549
10	2	8	1,194	1,151	1,129	0,592	1,834
10	2	9	1,473	1,424	1,397	0,761	2,236
10	2	10	1,948	1,879	1,831	1,021	3,017
10	3	1	0,317	0,281	0,264	0,051	0,608
10	3	2	0,46	0,433	0,421	0,16	0,768
10	3	3	0,58	0,557	0,548	0,263	0,902
10	3	4	0,692	0,671	0,663	0,36	1,027
10	3	5	0,804	0,784	0,775	0,455	1,155
10	3	6	0,92	0,9	0,891	0,551	1,294
10	3	7	1,047	1,027	1,017	0,654	1,454
10	3	8	1,195	1,174	1,162	0,765	1,644
10	3	9	1,388	1,363	1,347	0,899	1,915
10	3	10	1,71	1,674	1,642	1,083	2,447
10	6	1	0,531	0,514	0,514	0,249	0,78
10	6	2	0,658	0,647	0,645	0,41	0,891
10	6	3	0,751	0,741	0,74	0,513	0,978

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
10	6	4	0,831	0,823	0,82	0,599	1,058
10	6	5	0,907	0,899	0,896	0,673	1,139
10	6	6	0,983	0,975	0,972	0,745	1,224
10	6	7	1,064	1,056	1,052	0,815	1,322
10	6	8	1,157	1,148	1,144	0,891	1,435
10	6	9	1,275	1,265	1,257	0,978	1,599
10	6	10	1,469	1,455	1,437	1,095	1,913
10	9	1	0,623	0,611	0,614	0,373	0,838
10	9	2	0,732	0,724	0,725	0,522	0,922
10	9	3	0,808	0,802	0,802	0,612	0,994
10	9	4	0,874	0,869	0,868	0,684	1,056
10	9	5	0,935	0,93	0,929	0,748	1,121
10	9	6	0,996	0,991	0,989	0,803	1,187
10	9	7	1,06	1,055	1,052	0,862	1,261
10	9	8	1,133	1,127	1,124	0,923	1,35
10	9	9	1,225	1,219	1,213	0,992	1,48
10	9	10	1,377	1,367	1,353	1,083	1,724
10	12	1	0,676	0,668	0,671	0,451	0,863
10	12	2	0,773	0,768	0,77	0,589	0,941
10	12	3	0,84	0,835	0,835	0,673	0,999
10	12	4	0,896	0,892	0,891	0,735	1,055
10	12	5	0,948	0,945	0,943	0,789	1,109
10	12	6	1	0,997	0,995	0,841	1,164
10	12	7	1,055	1,051	1,049	0,888	1,227
10	12	8	1,117	1,113	1,11	0,94	1,304
10	12	9	1,196	1,191	1,186	1,001	1,412
10	12	10	1,321	1,314	1,302	1,078	1,616
10	16	1	0,721	0,715	0,718	0,525	0,89
10	16	2	0,806	0,803	0,804	0,649	0,953
10	16	3	0,865	0,862	0,862	0,72	1,003
10	16	4	0,913	0,911	0,911	0,774	1,05
10	16	5	0,959	0,956	0,955	0,822	1,096
10	16	6	1,003	1	0,999	0,864	1,143
10	16	7	1,05	1,048	1,046	0,907	1,198
10	16	8	1,104	1,101	1,098	0,952	1,264
10	16	9	1,17	1,167	1,162	1,005	1,353
10	16	10	1,277	1,272	1,263	1,07	1,526
10	20	1	0,753	0,748	0,752	0,571	0,902
10	20	2	0,83	0,827	0,828	0,683	0,959
10	20	3	0,881	0,879	0,879	0,754	1,004
10	20	4	0,925	0,922	0,922	0,8	1,045
10	20	5	0,965	0,963	0,962	0,844	1,086
10	20	6	1,005	1,003	1,001	0,882	1,13
10	20	7	1,047	1,045	1,043	0,92	1,178
10	20	8	1,094	1,092	1,09	0,961	1,234

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
10	20	9	1,152	1,15	1,146	1,007	1,312
10	20	10	1,247	1,243	1,234	1,064	1,464
10	25	1	0,779	0,775	0,779	0,615	0,913
10	25	2	0,849	0,847	0,848	0,723	0,964
10	25	3	0,896	0,894	0,895	0,783	1,008
10	25	4	0,935	0,933	0,933	0,825	1,042
10	25	5	0,97	0,969	0,968	0,862	1,077
10	25	6	1,006	1,004	1,003	0,897	1,114
10	25	7	1,042	1,04	1,039	0,932	1,157
10	25	8	1,084	1,082	1,08	0,965	1,207
10	25	9	1,136	1,134	1,131	1,008	1,28
10	25	10	1,219	1,215	1,209	1,057	1,412
11	2	1	0,144	0,105	0,076	0,003	0,364
11	2	2	0,252	0,212	0,186	0,029	0,541
11	2	3	0,362	0,323	0,3	0,076	0,7
11	2	4	0,477	0,439	0,418	0,138	0,859
11	2	5	0,599	0,561	0,542	0,212	1,019
11	2	6	0,73	0,693	0,675	0,299	1,19
11	2	7	0,876	0,838	0,821	0,396	1,379
11	2	8	1,042	1,004	0,987	0,511	1,599
11	2	9	1,245	1,205	1,186	0,649	1,875
11	2	10	1,519	1,472	1,446	0,821	2,273
11	2	11	1,986	1,92	1,871	1,07	3,041
11	3	1	0,302	0,267	0,251	0,048	0,58
11	3	2	0,437	0,411	0,4	0,152	0,731
11	3	3	0,55	0,527	0,518	0,25	0,854
11	3	4	0,653	0,633	0,625	0,341	0,971
11	3	5	0,755	0,736	0,728	0,43	1,086
11	3	6	0,858	0,84	0,832	0,516	1,204
11	3	7	0,968	0,949	0,942	0,607	1,335
11	3	8	1,089	1,07	1,061	0,703	1,486
11	3	9	1,232	1,212	1,201	0,813	1,677
11	3	10	1,421	1,398	1,382	0,941	1,947
11	3	11	1,738	1,704	1,671	1,121	2,472
11	6	1	0,519	0,502	0,502	0,244	0,759
11	6	2	0,641	0,63	0,629	0,398	0,862
11	6	3	0,728	0,72	0,719	0,5	0,944
11	6	4	0,803	0,796	0,794	0,582	1,019
11	6	5	0,873	0,866	0,864	0,652	1,092
11	6	6	0,942	0,935	0,932	0,719	1,169
11	6	7	1,013	1,006	1,003	0,782	1,249
11	6	8	1,091	1,083	1,079	0,85	1,34
11	6	9	1,18	1,173	1,167	0,92	1,459
11	6	10	1,296	1,286	1,277	1,002	1,617
11	6	11	1,486	1,471	1,453	1,119	1,933

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
11	9	1	0,614	0,603	0,605	0,37	0,821
11	9	2	0,72	0,713	0,712	0,518	0,905
11	9	3	0,793	0,787	0,787	0,607	0,973
11	9	4	0,854	0,849	0,847	0,673	1,03
11	9	5	0,91	0,905	0,905	0,732	1,085
11	9	6	0,965	0,96	0,959	0,787	1,143
11	9	7	1,021	1,017	1,015	0,838	1,207
11	9	8	1,082	1,077	1,074	0,892	1,278
11	9	9	1,151	1,147	1,143	0,95	1,365
11	9	10	1,241	1,235	1,229	1,017	1,493
11	9	11	1,387	1,378	1,365	1,102	1,727
11	12	1	0,667	0,659	0,663	0,445	0,849
11	12	2	0,761	0,756	0,756	0,582	0,923
11	12	3	0,826	0,822	0,822	0,662	0,98
11	12	4	0,879	0,875	0,875	0,726	1,029
11	12	5	0,927	0,923	0,923	0,774	1,078
11	12	6	0,974	0,971	0,97	0,821	1,128
11	12	7	1,022	1,019	1,018	0,863	1,179
11	12	8	1,074	1,071	1,069	0,91	1,241
11	12	9	1,134	1,13	1,128	0,961	1,314
11	12	10	1,209	1,205	1,2	1,018	1,419
11	12	11	1,333	1,326	1,314	1,093	1,624
11	16	1	0,713	0,707	0,712	0,514	0,875
11	16	2	0,796	0,793	0,795	0,643	0,937
11	16	3	0,852	0,849	0,85	0,71	0,983
11	16	4	0,898	0,896	0,896	0,764	1,027
11	16	5	0,94	0,938	0,938	0,808	1,069
11	16	6	0,981	0,979	0,978	0,85	1,112
11	16	7	1,022	1,019	1,018	0,888	1,156
11	16	8	1,066	1,063	1,062	0,928	1,206
11	16	9	1,117	1,114	1,111	0,971	1,271
11	16	10	1,181	1,178	1,174	1,018	1,36
11	16	11	1,286	1,281	1,272	1,081	1,53
11	20	1	0,747	0,743	0,746	0,571	0,895
11	20	2	0,821	0,818	0,82	0,68	0,948
11	20	3	0,872	0,869	0,87	0,745	0,988
11	20	4	0,913	0,911	0,911	0,792	1,029
11	20	5	0,95	0,948	0,949	0,833	1,064
11	20	6	0,986	0,984	0,984	0,867	1,103
11	20	7	1,022	1,021	1,02	0,902	1,142
11	20	8	1,062	1,06	1,059	0,939	1,186
11	20	9	1,106	1,104	1,102	0,975	1,242
11	20	10	1,163	1,16	1,156	1,018	1,322
11	20	11	1,255	1,251	1,243	1,077	1,471
11	25	1	0,773	0,769	0,774	0,608	0,904

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
11	25	2	0,841	0,838	0,84	0,715	0,953
11	25	3	0,885	0,884	0,884	0,771	0,992
11	25	4	0,922	0,92	0,92	0,815	1,026
11	25	5	0,955	0,954	0,953	0,852	1,059
11	25	6	0,987	0,986	0,985	0,883	1,092
11	25	7	1,019	1,018	1,016	0,916	1,127
11	25	8	1,054	1,052	1,05	0,946	1,168
11	25	9	1,094	1,092	1,09	0,979	1,215
11	25	10	1,144	1,142	1,139	1,016	1,283
11	25	11	1,227	1,223	1,214	1,068	1,423
12	2	1	0,133	0,097	0,071	0,003	0,338
12	2	2	0,233	0,196	0,172	0,026	0,501
12	2	3	0,334	0,297	0,275	0,068	0,651
12	2	4	0,438	0,402	0,383	0,124	0,793
12	2	5	0,547	0,512	0,494	0,191	0,936
12	2	6	0,664	0,629	0,612	0,267	1,088
12	2	7	0,791	0,756	0,74	0,353	1,254
12	2	8	0,932	0,897	0,881	0,453	1,438
12	2	9	1,095	1,059	1,042	0,566	1,65
12	2	10	1,294	1,256	1,236	0,7	1,92
12	2	11	1,562	1,518	1,491	0,871	2,308
12	2	12	2,024	1,961	1,914	1,117	3,079
12	3	1	0,288	0,255	0,239	0,046	0,553
12	3	2	0,417	0,392	0,382	0,145	0,695
12	3	3	0,523	0,502	0,494	0,238	0,81
12	3	4	0,62	0,601	0,593	0,322	0,919
12	3	5	0,714	0,696	0,69	0,406	1,026
12	3	6	0,808	0,79	0,784	0,487	1,133
12	3	7	0,905	0,888	0,882	0,57	1,247
12	3	8	1,009	0,992	0,985	0,653	1,372
12	3	9	1,126	1,109	1,1	0,747	1,518
12	3	10	1,265	1,247	1,236	0,85	1,702
12	3	11	1,449	1,427	1,411	0,98	1,965
12	3	12	1,759	1,726	1,696	1,152	2,476
12	6	1	0,508	0,492	0,492	0,24	0,742
12	6	2	0,626	0,615	0,615	0,39	0,843
12	6	3	0,71	0,701	0,7	0,488	0,921
12	6	4	0,781	0,773	0,772	0,565	0,99
12	6	5	0,846	0,839	0,837	0,632	1,056
12	6	6	0,91	0,903	0,902	0,694	1,123
12	6	7	0,974	0,968	0,965	0,753	1,195
12	6	8	1,041	1,034	1,032	0,813	1,27
12	6	9	1,115	1,108	1,105	0,878	1,36
12	6	10	1,201	1,193	1,188	0,946	1,472
12	6	11	1,314	1,305	1,296	1,026	1,633

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
12	6	12	1,5	1,486	1,467	1,14	1,933
12	9	1	0,603	0,592	0,595	0,362	0,807
12	9	2	0,706	0,699	0,7	0,507	0,888
12	9	3	0,776	0,771	0,771	0,595	0,949
12	9	4	0,835	0,83	0,83	0,66	1,004
12	9	5	0,887	0,883	0,882	0,716	1,058
12	9	6	0,938	0,934	0,933	0,766	1,109
12	9	7	0,989	0,985	0,983	0,815	1,163
12	9	8	1,042	1,038	1,035	0,864	1,223
12	9	9	1,099	1,095	1,092	0,914	1,291
12	9	10	1,167	1,162	1,159	0,966	1,377
12	9	11	1,255	1,25	1,243	1,033	1,499
12	9	12	1,4	1,391	1,378	1,12	1,74
12	12	1	0,658	0,65	0,654	0,441	0,837
12	12	2	0,75	0,745	0,747	0,576	0,909
12	12	3	0,813	0,809	0,809	0,654	0,963
12	12	4	0,863	0,86	0,86	0,713	1,008
12	12	5	0,909	0,906	0,905	0,763	1,053
12	12	6	0,953	0,95	0,948	0,806	1,099
12	12	7	0,996	0,993	0,992	0,85	1,146
12	12	8	1,042	1,039	1,037	0,891	1,199
12	12	9	1,091	1,088	1,086	0,933	1,256
12	12	10	1,149	1,145	1,142	0,98	1,329
12	12	11	1,222	1,218	1,213	1,034	1,433
12	12	12	1,343	1,336	1,324	1,105	1,634
12	16	1	0,708	0,702	0,707	0,515	0,866
12	16	2	0,789	0,785	0,787	0,636	0,927
12	16	3	0,842	0,839	0,84	0,701	0,972
12	16	4	0,885	0,883	0,884	0,753	1,01
12	16	5	0,925	0,922	0,923	0,796	1,047
12	16	6	0,962	0,96	0,959	0,836	1,084
12	16	7	0,999	0,997	0,997	0,872	1,123
12	16	8	1,038	1,035	1,034	0,908	1,169
12	16	9	1,08	1,077	1,077	0,944	1,218
12	16	10	1,128	1,126	1,124	0,984	1,279
12	16	11	1,191	1,188	1,184	1,031	1,369
12	16	12	1,295	1,29	1,279	1,093	1,543
12	20	1	0,739	0,734	0,737	0,567	0,884
12	20	2	0,812	0,809	0,811	0,676	0,936
12	20	3	0,86	0,858	0,859	0,738	0,976
12	20	4	0,899	0,897	0,897	0,779	1,011
12	20	5	0,935	0,933	0,932	0,821	1,046
12	20	6	0,968	0,967	0,966	0,853	1,08
12	20	7	1,001	1	0,999	0,89	1,115
12	20	8	1,036	1,034	1,032	0,922	1,156

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
12	20	9	1,073	1,071	1,07	0,951	1,197
12	20	10	1,116	1,114	1,112	0,989	1,248
12	20	11	1,17	1,168	1,164	1,028	1,326
12	20	12	1,262	1,258	1,249	1,085	1,475
12	25	1	0,768	0,764	0,768	0,611	0,896
12	25	2	0,834	0,832	0,834	0,713	0,944
12	25	3	0,879	0,877	0,877	0,769	0,983
12	25	4	0,913	0,912	0,912	0,811	1,013
12	25	5	0,945	0,943	0,942	0,845	1,044
12	25	6	0,974	0,973	0,972	0,874	1,075
12	25	7	1,003	1,002	1,001	0,903	1,106
12	25	8	1,033	1,032	1,031	0,931	1,138
12	25	9	1,066	1,065	1,063	0,961	1,178
12	25	10	1,104	1,103	1,101	0,993	1,224
12	25	11	1,152	1,15	1,147	1,027	1,289
12	25	12	1,233	1,23	1,223	1,079	1,42
13	2	1	0,124	0,09	0,065	0,002	0,314
13	2	2	0,217	0,182	0,159	0,024	0,468
13	2	3	0,31	0,276	0,254	0,064	0,605
13	2	4	0,406	0,372	0,353	0,115	0,737
13	2	5	0,505	0,472	0,455	0,175	0,868
13	2	6	0,611	0,578	0,562	0,244	1,003
13	2	7	0,723	0,691	0,677	0,321	1,146
13	2	8	0,846	0,814	0,8	0,405	1,302
13	2	9	0,982	0,95	0,935	0,505	1,48
13	2	10	1,142	1,108	1,092	0,617	1,691
13	2	11	1,336	1,3	1,282	0,748	1,954
13	2	12	1,6	1,558	1,532	0,918	2,345
13	2	13	2,053	1,993	1,945	1,161	3,09
13	3	1	0,278	0,246	0,23	0,044	0,536
13	3	2	0,401	0,376	0,367	0,139	0,669
13	3	3	0,501	0,481	0,473	0,228	0,776
13	3	4	0,592	0,574	0,567	0,309	0,878
13	3	5	0,68	0,663	0,657	0,385	0,976
13	3	6	0,766	0,75	0,744	0,462	1,071
13	3	7	0,854	0,838	0,832	0,539	1,173
13	3	8	0,946	0,931	0,925	0,615	1,28
13	3	9	1,047	1,031	1,024	0,699	1,402
13	3	10	1,16	1,144	1,136	0,789	1,548
13	3	11	1,296	1,279	1,269	0,891	1,727
13	3	12	1,478	1,456	1,441	1,01	1,991
13	3	13	1,784	1,752	1,72	1,183	2,501
13	6	1	0,499	0,483	0,484	0,236	0,726
13	6	2	0,614	0,603	0,603	0,386	0,824
13	6	3	0,694	0,686	0,685	0,481	0,898

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
13	6	4	0,762	0,755	0,753	0,555	0,965
13	6	5	0,824	0,818	0,816	0,619	1,025
13	6	6	0,883	0,876	0,875	0,676	1,084
13	6	7	0,941	0,935	0,933	0,731	1,151
13	6	8	1,001	0,995	0,993	0,786	1,219
13	6	9	1,065	1,059	1,056	0,842	1,296
13	6	10	1,136	1,13	1,125	0,903	1,381
13	6	11	1,22	1,213	1,206	0,967	1,492
13	6	12	1,329	1,321	1,312	1,046	1,642
13	6	13	1,514	1,501	1,483	1,155	1,95
13	9	1	0,596	0,586	0,589	0,36	0,795
13	9	2	0,696	0,69	0,691	0,5	0,871
13	9	3	0,764	0,758	0,759	0,585	0,93
13	9	4	0,819	0,815	0,815	0,649	0,983
13	9	5	0,87	0,866	0,865	0,703	1,035
13	9	6	0,918	0,914	0,913	0,752	1,083
13	9	7	0,964	0,96	0,959	0,796	1,129
13	9	8	1,011	1,008	1,006	0,841	1,182
13	9	9	1,062	1,058	1,056	0,887	1,24
13	9	10	1,117	1,113	1,111	0,935	1,307
13	9	11	1,182	1,177	1,173	0,987	1,392
13	9	12	1,267	1,262	1,256	1,051	1,508
13	9	13	1,41	1,401	1,386	1,132	1,756
13	12	1	0,651	0,643	0,648	0,438	0,828
13	12	2	0,741	0,736	0,738	0,569	0,895
13	12	3	0,801	0,797	0,797	0,645	0,946
13	12	4	0,849	0,846	0,847	0,7	0,99
13	12	5	0,893	0,89	0,89	0,75	1,033
13	12	6	0,933	0,931	0,93	0,791	1,075
13	12	7	0,973	0,97	0,969	0,832	1,117
13	12	8	1,014	1,012	1,011	0,868	1,16
13	12	9	1,057	1,054	1,052	0,906	1,209
13	12	10	1,104	1,101	1,098	0,948	1,265
13	12	11	1,159	1,155	1,152	0,993	1,334
13	12	12	1,23	1,226	1,221	1,047	1,435
13	12	13	1,35	1,343	1,332	1,118	1,633
13	16	1	0,7	0,694	0,699	0,508	0,856
13	16	2	0,779	0,775	0,777	0,63	0,911
13	16	3	0,83	0,828	0,828	0,696	0,956
13	16	4	0,873	0,87	0,871	0,746	0,992
13	16	5	0,91	0,908	0,907	0,787	1,029
13	16	6	0,946	0,944	0,942	0,826	1,064
13	16	7	0,98	0,978	0,977	0,86	1,1
13	16	8	1,015	1,013	1,011	0,894	1,138
13	16	9	1,051	1,049	1,047	0,926	1,179

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
13	16	10	1,091	1,089	1,087	0,96	1,229
13	16	11	1,139	1,137	1,135	0,999	1,288
13	16	12	1,201	1,198	1,194	1,045	1,375
13	16	13	1,302	1,297	1,289	1,105	1,543
13	20	1	0,732	0,728	0,732	0,561	0,873
13	20	2	0,804	0,801	0,803	0,668	0,924
13	20	3	0,851	0,849	0,849	0,731	0,965
13	20	4	0,888	0,887	0,887	0,777	0,996
13	20	5	0,922	0,92	0,92	0,812	1,029
13	20	6	0,953	0,951	0,951	0,847	1,059
13	20	7	0,983	0,982	0,981	0,877	1,089
13	20	8	1,015	1,013	1,012	0,907	1,126
13	20	9	1,048	1,046	1,044	0,938	1,166
13	20	10	1,084	1,082	1,081	0,969	1,206
13	20	11	1,126	1,124	1,121	1,002	1,258
13	20	12	1,18	1,178	1,174	1,043	1,334
13	20	13	1,27	1,266	1,258	1,094	1,484
13	25	1	0,761	0,758	0,761	0,6	0,89
13	25	2	0,827	0,824	0,826	0,705	0,936
13	25	3	0,869	0,867	0,868	0,759	0,968
13	25	4	0,902	0,901	0,901	0,8	0,998
13	25	5	0,932	0,931	0,931	0,834	1,027
13	25	6	0,96	0,959	0,959	0,862	1,055
13	25	7	0,987	0,986	0,986	0,891	1,083
13	25	8	1,015	1,013	1,012	0,917	1,113
13	25	9	1,044	1,042	1,041	0,943	1,147
13	25	10	1,075	1,074	1,073	0,972	1,186
13	25	11	1,112	1,11	1,108	1,002	1,233
13	25	12	1,159	1,157	1,154	1,038	1,3
13	25	13	1,238	1,235	1,228	1,084	1,43
14	2	1	0,116	0,084	0,061	0,002	0,293
14	2	2	0,202	0,169	0,147	0,022	0,437
14	2	3	0,288	0,256	0,237	0,059	0,566
14	2	4	0,377	0,345	0,327	0,105	0,685
14	2	5	0,468	0,437	0,421	0,161	0,805
14	2	6	0,565	0,534	0,518	0,224	0,931
14	2	7	0,666	0,636	0,622	0,293	1,062
14	2	8	0,775	0,745	0,731	0,371	1,202
14	2	9	0,895	0,865	0,851	0,457	1,351
14	2	10	1,029	0,998	0,984	0,554	1,524
14	2	11	1,184	1,152	1,136	0,662	1,726
14	2	12	1,375	1,341	1,323	0,796	1,989
14	2	13	1,633	1,593	1,567	0,959	2,369
14	2	14	2,081	2,023	1,974	1,202	3,123
14	3	1	0,267	0,237	0,222	0,043	0,515

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
14	3	2	0,385	0,362	0,352	0,135	0,642
14	3	3	0,481	0,461	0,453	0,219	0,745
14	3	4	0,568	0,55	0,544	0,297	0,842
14	3	5	0,65	0,634	0,628	0,371	0,932
14	3	6	0,73	0,714	0,709	0,442	1,022
14	3	7	0,811	0,796	0,79	0,512	1,112
14	3	8	0,894	0,88	0,874	0,583	1,209
14	3	9	0,984	0,969	0,962	0,659	1,315
14	3	10	1,08	1,065	1,057	0,737	1,431
14	3	11	1,19	1,175	1,166	0,821	1,575
14	3	12	1,322	1,305	1,295	0,922	1,751
14	3	13	1,498	1,478	1,463	1,041	2,001
14	3	14	1,8	1,769	1,736	1,208	2,508
14	6	1	0,49	0,475	0,475	0,234	0,716
14	6	2	0,602	0,592	0,592	0,377	0,806
14	6	3	0,68	0,672	0,671	0,47	0,878
14	6	4	0,745	0,739	0,738	0,543	0,941
14	6	5	0,804	0,798	0,797	0,606	1,001
14	6	6	0,86	0,854	0,852	0,662	1,057
14	6	7	0,914	0,908	0,906	0,715	1,114
14	6	8	0,969	0,963	0,961	0,766	1,174
14	6	9	1,027	1,021	1,018	0,816	1,241
14	6	10	1,088	1,082	1,078	0,872	1,314
14	6	11	1,156	1,15	1,145	0,927	1,393
14	6	12	1,237	1,23	1,225	0,992	1,5
14	6	13	1,345	1,336	1,328	1,068	1,653
14	6	14	1,525	1,512	1,495	1,174	1,952
14	9	1	0,588	0,578	0,582	0,355	0,783
14	9	2	0,685	0,679	0,681	0,494	0,857
14	9	3	0,751	0,746	0,745	0,578	0,914
14	9	4	0,805	0,801	0,801	0,643	0,963
14	9	5	0,852	0,849	0,848	0,695	1,01
14	9	6	0,897	0,894	0,892	0,74	1,056
14	9	7	0,941	0,938	0,936	0,783	1,101
14	9	8	0,984	0,981	0,979	0,823	1,145
14	9	9	1,03	1,026	1,024	0,867	1,195
14	9	10	1,079	1,075	1,074	0,911	1,251
14	9	11	1,132	1,129	1,127	0,954	1,317
14	9	12	1,196	1,192	1,188	1,004	1,4
14	9	13	1,28	1,274	1,269	1,062	1,519
14	9	14	1,42	1,412	1,399	1,144	1,748
14	12	1	0,646	0,638	0,642	0,438	0,819
14	12	2	0,732	0,728	0,729	0,565	0,883
14	12	3	0,79	0,787	0,787	0,641	0,932
14	12	4	0,837	0,834	0,834	0,696	0,973

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
14	12	5	0,878	0,875	0,875	0,74	1,014
14	12	6	0,917	0,914	0,914	0,778	1,052
14	12	7	0,954	0,951	0,951	0,817	1,09
14	12	8	0,991	0,989	0,988	0,853	1,129
14	12	9	1,03	1,028	1,026	0,888	1,171
14	12	10	1,071	1,068	1,066	0,923	1,217
14	12	11	1,116	1,114	1,112	0,965	1,272
14	12	12	1,17	1,167	1,164	1,007	1,342
14	12	13	1,24	1,236	1,231	1,058	1,445
14	12	14	1,358	1,352	1,34	1,129	1,64
14	16	1	0,693	0,688	0,692	0,508	0,846
14	16	2	0,771	0,768	0,769	0,624	0,902
14	16	3	0,822	0,819	0,821	0,688	0,943
14	16	4	0,862	0,86	0,861	0,737	0,979
14	16	5	0,898	0,896	0,895	0,781	1,015
14	16	6	0,932	0,93	0,929	0,815	1,046
14	16	7	0,964	0,962	0,961	0,848	1,08
14	16	8	0,997	0,995	0,994	0,881	1,115
14	16	9	1,029	1,028	1,027	0,912	1,151
14	16	10	1,065	1,063	1,062	0,944	1,192
14	16	11	1,103	1,101	1,1	0,975	1,238
14	16	12	1,15	1,147	1,145	1,013	1,296
14	16	13	1,211	1,208	1,203	1,057	1,386
14	16	14	1,308	1,303	1,295	1,113	1,545
14	20	1	0,729	0,724	0,728	0,559	0,865
14	20	2	0,798	0,796	0,798	0,666	0,917
14	20	3	0,844	0,842	0,843	0,725	0,953
14	20	4	0,88	0,878	0,878	0,77	0,986
14	20	5	0,912	0,911	0,91	0,807	1,016
14	20	6	0,943	0,941	0,94	0,839	1,045
14	20	7	0,971	0,97	0,969	0,869	1,073
14	20	8	0,999	0,998	0,997	0,894	1,104
14	20	9	1,028	1,027	1,026	0,922	1,135
14	20	10	1,06	1,058	1,057	0,952	1,17
14	20	11	1,094	1,092	1,091	0,98	1,213
14	20	12	1,134	1,132	1,131	1,012	1,262
14	20	13	1,187	1,184	1,181	1,048	1,341
14	20	14	1,274	1,27	1,26	1,104	1,489
14	25	1	0,756	0,752	0,756	0,603	0,88
14	25	2	0,82	0,818	0,819	0,7	0,926
14	25	3	0,861	0,86	0,861	0,756	0,959
14	25	4	0,894	0,893	0,893	0,795	0,988
14	25	5	0,923	0,922	0,922	0,827	1,015
14	25	6	0,949	0,948	0,948	0,856	1,042
14	25	7	0,975	0,974	0,974	0,881	1,067

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
14	25	8	1,001	1	0,999	0,907	1,094
14	25	9	1,027	1,026	1,025	0,934	1,125
14	25	10	1,054	1,053	1,051	0,958	1,153
14	25	11	1,084	1,083	1,082	0,985	1,187
14	25	12	1,121	1,119	1,118	1,016	1,236
14	25	13	1,167	1,165	1,163	1,048	1,302
14	25	14	1,244	1,241	1,233	1,093	1,43
15	2	1	0,108	0,078	0,056	0,002	0,274
15	2	2	0,189	0,158	0,137	0,021	0,412
15	2	3	0,269	0,239	0,22	0,055	0,53
15	2	4	0,351	0,322	0,304	0,098	0,642
15	2	5	0,436	0,407	0,391	0,149	0,755
15	2	6	0,524	0,495	0,481	0,206	0,867
15	2	7	0,617	0,589	0,575	0,27	0,984
15	2	8	0,715	0,687	0,673	0,34	1,108
15	2	9	0,821	0,793	0,781	0,418	1,243
15	2	10	0,937	0,909	0,896	0,501	1,394
15	2	11	1,069	1,04	1,027	0,597	1,561
15	2	12	1,222	1,192	1,178	0,706	1,763
15	2	13	1,409	1,376	1,358	0,832	2,018
15	2	14	1,664	1,625	1,602	0,996	2,395
15	2	15	2,11	2,052	2,005	1,23	3,147
15	3	1	0,257	0,228	0,214	0,041	0,495
15	3	2	0,371	0,349	0,34	0,129	0,62
15	3	3	0,463	0,444	0,437	0,211	0,721
15	3	4	0,546	0,529	0,523	0,285	0,811
15	3	5	0,624	0,608	0,602	0,354	0,896
15	3	6	0,7	0,685	0,679	0,422	0,979
15	3	7	0,775	0,761	0,755	0,488	1,066
15	3	8	0,852	0,838	0,833	0,557	1,152
15	3	9	0,932	0,918	0,913	0,625	1,243
15	3	10	1,017	1,003	0,997	0,696	1,345
15	3	11	1,111	1,097	1,09	0,772	1,461
15	3	12	1,218	1,204	1,195	0,859	1,596
15	3	13	1,348	1,332	1,321	0,953	1,768
15	3	14	1,523	1,503	1,487	1,068	2,024
15	3	15	1,823	1,793	1,762	1,237	2,524
15	6	1	0,481	0,466	0,467	0,229	0,701
15	6	2	0,591	0,581	0,58	0,374	0,789
15	6	3	0,666	0,658	0,658	0,463	0,856
15	6	4	0,729	0,723	0,722	0,534	0,917
15	6	5	0,785	0,779	0,779	0,593	0,973
15	6	6	0,838	0,833	0,83	0,647	1,027
15	6	7	0,889	0,884	0,882	0,697	1,08
15	6	8	0,941	0,935	0,934	0,746	1,135

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
15	6	9	0,992	0,987	0,985	0,793	1,193
15	6	10	1,047	1,041	1,039	0,843	1,255
15	6	11	1,105	1,1	1,097	0,895	1,325
15	6	12	1,172	1,166	1,162	0,949	1,408
15	6	13	1,251	1,244	1,238	1,01	1,509
15	6	14	1,357	1,348	1,34	1,084	1,663
15	6	15	1,536	1,523	1,506	1,183	1,959
15	9	1	0,582	0,572	0,575	0,355	0,77
15	9	2	0,677	0,671	0,672	0,491	0,846
15	9	3	0,741	0,736	0,736	0,57	0,902
15	9	4	0,793	0,789	0,789	0,633	0,949
15	9	5	0,839	0,835	0,834	0,683	0,991
15	9	6	0,882	0,878	0,877	0,728	1,032
15	9	7	0,923	0,919	0,918	0,769	1,075
15	9	8	0,963	0,96	0,959	0,809	1,117
15	9	9	1,005	1,002	1,001	0,849	1,162
15	9	10	1,048	1,044	1,043	0,888	1,211
15	9	11	1,094	1,09	1,089	0,928	1,266
15	9	12	1,146	1,142	1,141	0,969	1,329
15	9	13	1,208	1,204	1,2	1,017	1,413
15	9	14	1,29	1,285	1,279	1,077	1,529
15	9	15	1,428	1,42	1,406	1,156	1,757
15	12	1	0,64	0,632	0,637	0,433	0,81
15	12	2	0,725	0,72	0,722	0,556	0,872
15	12	3	0,781	0,778	0,778	0,63	0,921
15	12	4	0,826	0,823	0,824	0,685	0,959
15	12	5	0,866	0,864	0,863	0,731	0,996
15	12	6	0,904	0,901	0,9	0,771	1,034
15	12	7	0,939	0,936	0,936	0,807	1,068
15	12	8	0,973	0,971	0,97	0,84	1,106
15	12	9	1,009	1,006	1,005	0,875	1,143
15	12	10	1,046	1,043	1,042	0,907	1,185
15	12	11	1,085	1,082	1,081	0,941	1,232
15	12	12	1,13	1,127	1,125	0,978	1,286
15	12	13	1,182	1,179	1,176	1,023	1,358
15	12	14	1,251	1,247	1,241	1,073	1,452
15	12	15	1,368	1,362	1,35	1,144	1,65
15	16	1	0,689	0,683	0,687	0,503	0,84
15	16	2	0,766	0,762	0,765	0,618	0,892
15	16	3	0,815	0,812	0,814	0,684	0,934
15	16	4	0,854	0,852	0,853	0,732	0,969
15	16	5	0,889	0,887	0,887	0,771	1,001
15	16	6	0,92	0,919	0,918	0,808	1,034
15	16	7	0,95	0,949	0,948	0,838	1,064
15	16	8	0,981	0,979	0,979	0,869	1,096

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
15	16	9	1,011	1,009	1,008	0,899	1,126
15	16	10	1,042	1,041	1,039	0,927	1,163
15	16	11	1,076	1,074	1,073	0,955	1,201
15	16	12	1,114	1,112	1,109	0,989	1,247
15	16	13	1,158	1,156	1,153	1,025	1,306
15	16	14	1,218	1,215	1,21	1,068	1,386
15	16	15	1,315	1,31	1,301	1,123	1,551
15	20	1	0,723	0,718	0,723	0,552	0,858
15	20	2	0,791	0,788	0,79	0,662	0,908
15	20	3	0,835	0,833	0,836	0,717	0,942
15	20	4	0,87	0,869	0,87	0,76	0,971
15	20	5	0,902	0,9	0,9	0,798	1,001
15	20	6	0,93	0,929	0,929	0,828	1,029
15	20	7	0,957	0,956	0,956	0,856	1,057
15	20	8	0,984	0,983	0,982	0,884	1,084
15	20	9	1,011	1,009	1,008	0,909	1,113
15	20	10	1,038	1,037	1,036	0,934	1,142
15	20	11	1,069	1,067	1,066	0,96	1,178
15	20	12	1,102	1,101	1,099	0,989	1,22
15	20	13	1,142	1,141	1,138	1,02	1,277
15	20	14	1,194	1,192	1,187	1,061	1,345
15	20	15	1,281	1,277	1,269	1,11	1,491
16	2	1	0,102	0,074	0,053	0,002	0,261
16	2	2	0,178	0,149	0,129	0,019	0,387
16	2	3	0,254	0,225	0,207	0,051	0,502
16	2	4	0,331	0,302	0,286	0,091	0,606
16	2	5	0,41	0,382	0,367	0,138	0,711
16	2	6	0,491	0,464	0,45	0,191	0,817
16	2	7	0,576	0,55	0,536	0,25	0,924
16	2	8	0,666	0,64	0,627	0,315	1,037
16	2	9	0,762	0,736	0,724	0,383	1,157
16	2	10	0,866	0,839	0,827	0,459	1,289
16	2	11	0,98	0,953	0,94	0,543	1,436
16	2	12	1,108	1,08	1,067	0,636	1,602
16	2	13	1,258	1,229	1,214	0,744	1,799
16	2	14	1,442	1,41	1,393	0,868	2,05
16	2	15	1,694	1,656	1,631	1,029	2,421
16	2	16	2,132	2,077	2,031	1,264	3,156
16	3	1	0,25	0,221	0,208	0,039	0,478
16	3	2	0,359	0,337	0,329	0,125	0,597
16	3	3	0,447	0,429	0,423	0,203	0,695
16	3	4	0,527	0,511	0,505	0,275	0,781
16	3	5	0,601	0,586	0,58	0,341	0,862
16	3	6	0,672	0,658	0,652	0,406	0,94
16	3	7	0,743	0,729	0,725	0,468	1,017

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
16	3	8	0,814	0,801	0,796	0,532	1,096
16	3	9	0,887	0,875	0,87	0,596	1,182
16	3	10	0,964	0,952	0,946	0,662	1,271
16	3	11	1,047	1,034	1,028	0,73	1,371
16	3	12	1,139	1,126	1,119	0,803	1,485
16	3	13	1,244	1,23	1,221	0,885	1,616
16	3	14	1,372	1,356	1,346	0,977	1,791
16	3	15	1,543	1,524	1,509	1,095	2,04
16	3	16	1,84	1,81	1,778	1,259	2,534
16	6	1	0,474	0,459	0,46	0,228	0,692
16	6	2	0,581	0,571	0,571	0,366	0,778
16	6	3	0,654	0,647	0,646	0,454	0,845
16	6	4	0,715	0,708	0,707	0,522	0,899
16	6	5	0,769	0,763	0,762	0,578	0,952
16	6	6	0,819	0,814	0,813	0,632	1,001
16	6	7	0,868	0,863	0,862	0,679	1,052
16	6	8	0,915	0,91	0,909	0,728	1,103
16	6	9	0,964	0,959	0,957	0,775	1,155
16	6	10	1,013	1,008	1,006	0,819	1,211
16	6	11	1,066	1,061	1,059	0,865	1,269
16	6	12	1,123	1,118	1,116	0,913	1,336
16	6	13	1,189	1,183	1,179	0,967	1,419
16	6	14	1,266	1,26	1,255	1,028	1,522
16	6	15	1,37	1,362	1,354	1,097	1,67
16	6	16	1,546	1,533	1,516	1,199	1,973
16	9	1	0,575	0,565	0,568	0,348	0,764
16	9	2	0,668	0,662	0,663	0,484	0,833
16	9	3	0,731	0,726	0,726	0,564	0,886
16	9	4	0,782	0,778	0,778	0,625	0,932
16	9	5	0,826	0,822	0,822	0,673	0,972
16	9	6	0,867	0,863	0,863	0,716	1,012
16	9	7	0,905	0,902	0,902	0,756	1,05
16	9	8	0,943	0,94	0,939	0,796	1,09
16	9	9	0,982	0,979	0,978	0,83	1,131
16	9	10	1,021	1,018	1,015	0,866	1,175
16	9	11	1,062	1,059	1,057	0,904	1,221
16	9	12	1,107	1,104	1,102	0,943	1,275
16	9	13	1,157	1,154	1,151	0,983	1,339
16	9	14	1,218	1,214	1,21	1,029	1,414
16	9	15	1,298	1,293	1,287	1,088	1,532
16	9	16	1,435	1,427	1,413	1,17	1,767
16	12	1	0,634	0,627	0,632	0,427	0,799
16	12	2	0,717	0,713	0,715	0,552	0,859
16	12	3	0,772	0,769	0,769	0,627	0,905
16	12	4	0,816	0,813	0,814	0,678	0,946

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
16	12	5	0,855	0,852	0,852	0,723	0,982
16	12	6	0,89	0,888	0,887	0,762	1,018
16	12	7	0,924	0,922	0,921	0,797	1,05
16	12	8	0,956	0,954	0,953	0,828	1,085
16	12	9	0,989	0,986	0,985	0,861	1,118
16	12	10	1,022	1,02	1,018	0,893	1,157
16	12	11	1,057	1,055	1,054	0,925	1,197
16	12	12	1,096	1,093	1,092	0,958	1,24
16	12	13	1,139	1,136	1,133	0,993	1,296
16	12	14	1,19	1,187	1,183	1,034	1,361
16	12	15	1,258	1,254	1,248	1,083	1,459
16	12	16	1,374	1,368	1,357	1,152	1,649
16	16	1	0,684	0,679	0,682	0,502	0,832
16	16	2	0,759	0,756	0,757	0,613	0,887
16	16	3	0,807	0,805	0,806	0,679	0,923
16	16	4	0,846	0,844	0,845	0,727	0,96
16	16	5	0,879	0,877	0,877	0,764	0,989
16	16	6	0,909	0,908	0,907	0,797	1,017
16	16	7	0,938	0,937	0,937	0,829	1,047
16	16	8	0,967	0,965	0,965	0,856	1,075
16	16	9	0,995	0,994	0,993	0,883	1,106
16	16	10	1,024	1,023	1,021	0,913	1,138
16	16	11	1,054	1,052	1,052	0,941	1,169
16	16	12	1,087	1,085	1,084	0,971	1,204
16	16	13	1,124	1,122	1,121	1,001	1,25
16	16	14	1,167	1,165	1,164	1,036	1,306
16	16	15	1,225	1,223	1,219	1,075	1,391
16	16	16	1,323	1,318	1,308	1,131	1,556
16	20	1	0,718	0,714	0,719	0,552	0,854
16	20	2	0,786	0,784	0,786	0,658	0,9
16	20	3	0,83	0,828	0,829	0,717	0,934
16	20	4	0,864	0,863	0,862	0,761	0,964
16	20	5	0,894	0,893	0,892	0,796	0,992
16	20	6	0,921	0,92	0,92	0,823	1,018
16	20	7	0,947	0,946	0,946	0,848	1,045
16	20	8	0,972	0,971	0,971	0,873	1,069
16	20	9	0,997	0,996	0,995	0,9	1,095
16	20	10	1,023	1,022	1,021	0,924	1,123
16	20	11	1,049	1,048	1,047	0,947	1,154
16	20	12	1,078	1,077	1,076	0,973	1,187
16	20	13	1,111	1,109	1,108	1,001	1,228
16	20	14	1,15	1,148	1,145	1,034	1,278
16	20	15	1,2	1,198	1,194	1,068	1,347
16	20	16	1,284	1,281	1,273	1,119	1,482
17	2	1	0,097	0,07	0,05	0,002	0,248

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
17	2	2	0,168	0,14	0,121	0,018	0,37
17	2	3	0,239	0,212	0,194	0,047	0,473
17	2	4	0,312	0,285	0,269	0,085	0,573
17	2	5	0,386	0,359	0,344	0,13	0,673
17	2	6	0,462	0,436	0,422	0,179	0,771
17	2	7	0,541	0,516	0,503	0,232	0,872
17	2	8	0,624	0,599	0,586	0,292	0,976
17	2	9	0,711	0,686	0,674	0,355	1,085
17	2	10	0,805	0,78	0,768	0,425	1,203
17	2	11	0,906	0,881	0,869	0,499	1,327
17	2	12	1,018	0,992	0,98	0,585	1,469
17	2	13	1,144	1,117	1,104	0,677	1,632
17	2	14	1,291	1,263	1,249	0,781	1,828
17	2	15	1,474	1,443	1,425	0,909	2,076
17	2	16	1,722	1,686	1,662	1,066	2,443
17	2	17	2,154	2,1	2,05	1,297	3,165
17	3	1	0,243	0,216	0,203	0,039	0,468
17	3	2	0,349	0,328	0,32	0,122	0,582
17	3	3	0,434	0,416	0,41	0,197	0,673
17	3	4	0,51	0,494	0,488	0,265	0,756
17	3	5	0,581	0,566	0,561	0,331	0,832
17	3	6	0,648	0,634	0,629	0,393	0,906
17	3	7	0,714	0,702	0,697	0,452	0,977
17	3	8	0,782	0,77	0,765	0,511	1,055
17	3	9	0,85	0,838	0,833	0,571	1,132
17	3	10	0,92	0,908	0,904	0,63	1,21
17	3	11	0,995	0,983	0,977	0,694	1,301
17	3	12	1,075	1,063	1,057	0,761	1,394
17	3	13	1,164	1,151	1,145	0,834	1,505
17	3	14	1,268	1,254	1,245	0,913	1,638
17	3	15	1,392	1,377	1,367	1,005	1,807
17	3	16	1,562	1,543	1,529	1,117	2,056
17	3	17	1,855	1,826	1,796	1,279	2,551
17	6	1	0,468	0,453	0,453	0,225	0,681
17	6	2	0,571	0,562	0,562	0,362	0,763
17	6	3	0,644	0,636	0,636	0,449	0,825
17	6	4	0,703	0,696	0,696	0,514	0,883
17	6	5	0,755	0,749	0,749	0,57	0,93
17	6	6	0,803	0,798	0,797	0,622	0,979
17	6	7	0,849	0,844	0,843	0,667	1,025
17	6	8	0,894	0,89	0,889	0,712	1,071
17	6	9	0,939	0,935	0,933	0,757	1,122
17	6	10	0,985	0,981	0,979	0,798	1,172
17	6	11	1,033	1,029	1,026	0,842	1,23
17	6	12	1,084	1,079	1,076	0,886	1,286

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
17	6	13	1,139	1,134	1,131	0,933	1,353
17	6	14	1,202	1,197	1,193	0,985	1,435
17	6	15	1,279	1,273	1,265	1,044	1,535
17	6	16	1,38	1,373	1,365	1,116	1,682
17	6	17	1,555	1,543	1,524	1,217	1,98
17	9	1	0,569	0,56	0,564	0,348	0,752
17	9	2	0,661	0,655	0,656	0,483	0,822
17	9	3	0,721	0,717	0,717	0,558	0,873
17	9	4	0,77	0,767	0,767	0,615	0,918
17	9	5	0,813	0,81	0,81	0,665	0,957
17	9	6	0,853	0,85	0,849	0,706	0,996
17	9	7	0,889	0,886	0,886	0,744	1,032
17	9	8	0,925	0,922	0,922	0,779	1,069
17	9	9	0,961	0,958	0,957	0,814	1,106
17	9	10	0,997	0,994	0,993	0,849	1,148
17	9	11	1,035	1,032	1,03	0,884	1,188
17	9	12	1,075	1,072	1,07	0,921	1,232
17	9	13	1,119	1,116	1,113	0,957	1,285
17	9	14	1,168	1,165	1,162	0,998	1,348
17	9	15	1,228	1,224	1,22	1,046	1,426
17	9	16	1,307	1,303	1,296	1,103	1,543
17	9	17	1,443	1,435	1,421	1,18	1,769
17	12	1	0,629	0,622	0,627	0,429	0,792
17	12	2	0,71	0,706	0,708	0,552	0,852
17	12	3	0,764	0,761	0,762	0,622	0,895
17	12	4	0,807	0,804	0,804	0,671	0,934
17	12	5	0,844	0,841	0,841	0,714	0,969
17	12	6	0,878	0,875	0,875	0,752	1,003
17	12	7	0,91	0,908	0,907	0,784	1,035
17	12	8	0,941	0,939	0,938	0,817	1,065
17	12	9	0,972	0,97	0,97	0,849	1,096
17	12	10	1,003	1,001	1	0,879	1,129
17	12	11	1,035	1,033	1,031	0,908	1,166
17	12	12	1,069	1,067	1,065	0,938	1,205
17	12	13	1,106	1,104	1,102	0,971	1,251
17	12	14	1,148	1,146	1,144	1,007	1,302
17	12	15	1,199	1,196	1,193	1,044	1,371
17	12	16	1,265	1,262	1,255	1,094	1,459
17	12	17	1,379	1,373	1,362	1,156	1,653
17	16	1	0,68	0,675	0,679	0,502	0,826
17	16	2	0,753	0,75	0,752	0,614	0,877
17	16	3	0,801	0,798	0,8	0,678	0,916
17	16	4	0,838	0,836	0,836	0,726	0,948
17	16	5	0,87	0,868	0,869	0,758	0,977
17	16	6	0,9	0,898	0,898	0,789	1,004

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
17	16	7	0,927	0,925	0,925	0,818	1,031
17	16	8	0,954	0,952	0,952	0,847	1,057
17	16	9	0,98	0,979	0,979	0,874	1,084
17	16	10	1,007	1,005	1,005	0,897	1,112
17	16	11	1,034	1,032	1,032	0,924	1,142
17	16	12	1,063	1,061	1,062	0,953	1,175
17	16	13	1,094	1,093	1,092	0,977	1,212
17	16	14	1,13	1,128	1,127	1,009	1,258
17	16	15	1,173	1,171	1,169	1,042	1,314
17	16	16	1,231	1,228	1,224	1,081	1,402
17	16	17	1,325	1,321	1,312	1,137	1,559
17	20	1	0,715	0,711	0,715	0,553	0,846
17	20	2	0,781	0,779	0,78	0,654	0,894
17	20	3	0,824	0,822	0,822	0,709	0,928
17	20	4	0,857	0,855	0,856	0,751	0,956
17	20	5	0,886	0,885	0,885	0,786	0,982
17	20	6	0,912	0,911	0,911	0,817	1,008
17	20	7	0,937	0,936	0,936	0,841	1,031
17	20	8	0,961	0,96	0,96	0,865	1,055
17	20	9	0,984	0,983	0,983	0,89	1,08
17	20	10	1,008	1,007	1,006	0,913	1,106
17	20	11	1,032	1,031	1,031	0,935	1,133
17	20	12	1,058	1,057	1,055	0,959	1,164
17	20	13	1,086	1,085	1,083	0,983	1,196
17	20	14	1,118	1,117	1,114	1,01	1,236
17	20	15	1,157	1,155	1,152	1,042	1,287
17	20	16	1,207	1,205	1,2	1,074	1,356
17	20	17	1,291	1,288	1,28	1,126	1,493
18	2	1	0,092	0,066	0,047	0,002	0,235
18	2	2	0,16	0,133	0,115	0,017	0,349
18	2	3	0,227	0,201	0,185	0,045	0,45
18	2	4	0,295	0,269	0,254	0,08	0,544
18	2	5	0,365	0,339	0,325	0,121	0,636
18	2	6	0,436	0,411	0,398	0,168	0,729
18	2	7	0,51	0,486	0,473	0,22	0,821
18	2	8	0,587	0,563	0,551	0,274	0,918
18	2	9	0,668	0,644	0,633	0,333	1,02
18	2	10	0,753	0,73	0,719	0,395	1,124
18	2	11	0,845	0,821	0,811	0,465	1,239
18	2	12	0,945	0,921	0,909	0,54	1,364
18	2	13	1,054	1,029	1,018	0,623	1,501
18	2	14	1,178	1,153	1,141	0,713	1,66
18	2	15	1,324	1,297	1,283	0,818	1,854
18	2	16	1,502	1,473	1,456	0,943	2,099
18	2	17	1,748	1,713	1,688	1,095	2,464

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
18	2	18	2,179	2,126	2,08	1,328	3,188
18	3	1	0,235	0,208	0,196	0,038	0,453
18	3	2	0,337	0,317	0,308	0,117	0,565
18	3	3	0,42	0,402	0,395	0,19	0,655
18	3	4	0,493	0,477	0,471	0,257	0,731
18	3	5	0,56	0,546	0,541	0,318	0,803
18	3	6	0,625	0,612	0,608	0,378	0,873
18	3	7	0,688	0,676	0,672	0,437	0,943
18	3	8	0,751	0,74	0,735	0,491	1,012
18	3	9	0,815	0,804	0,8	0,548	1,084
18	3	10	0,881	0,87	0,865	0,605	1,159
18	3	11	0,949	0,938	0,933	0,666	1,238
18	3	12	1,022	1,01	1,005	0,727	1,323
18	3	13	1,1	1,088	1,083	0,792	1,415
18	3	14	1,187	1,175	1,168	0,864	1,528
18	3	15	1,289	1,276	1,267	0,941	1,657
18	3	16	1,412	1,398	1,387	1,03	1,828
18	3	17	1,58	1,562	1,547	1,14	2,073
18	3	18	1,872	1,844	1,811	1,301	2,567
18	6	1	0,462	0,447	0,448	0,22	0,669
18	6	2	0,564	0,555	0,556	0,357	0,752
18	6	3	0,635	0,628	0,627	0,442	0,813
18	6	4	0,692	0,686	0,685	0,508	0,867
18	6	5	0,743	0,738	0,738	0,565	0,913
18	6	6	0,789	0,784	0,783	0,612	0,96
18	6	7	0,833	0,828	0,827	0,657	1,007
18	6	8	0,876	0,872	0,87	0,7	1,052
18	6	9	0,919	0,914	0,912	0,742	1,1
18	6	10	0,961	0,957	0,954	0,785	1,143
18	6	11	1,005	1,001	0,997	0,826	1,192
18	6	12	1,051	1,047	1,044	0,865	1,244
18	6	13	1,1	1,096	1,093	0,907	1,302
18	6	14	1,154	1,15	1,147	0,953	1,367
18	6	15	1,217	1,212	1,208	1,002	1,445
18	6	16	1,292	1,286	1,281	1,062	1,545
18	6	17	1,394	1,386	1,377	1,132	1,697
18	6	18	1,567	1,555	1,536	1,226	1,985
18	9	1	0,564	0,554	0,558	0,345	0,746
18	9	2	0,654	0,648	0,65	0,475	0,813
18	9	3	0,714	0,709	0,71	0,554	0,862
18	9	4	0,762	0,758	0,758	0,61	0,904
18	9	5	0,803	0,8	0,8	0,657	0,943
18	9	6	0,841	0,837	0,837	0,696	0,98
18	9	7	0,876	0,873	0,872	0,732	1,013
18	9	8	0,911	0,908	0,907	0,767	1,051

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
18	9	9	0,945	0,942	0,941	0,803	1,087
18	9	10	0,979	0,976	0,975	0,835	1,123
18	9	11	1,014	1,011	1,01	0,869	1,158
18	9	12	1,051	1,048	1,046	0,903	1,2
18	9	13	1,09	1,087	1,085	0,938	1,244
18	9	14	1,132	1,129	1,128	0,975	1,294
18	9	15	1,181	1,178	1,174	1,015	1,356
18	9	16	1,239	1,236	1,232	1,058	1,437
18	9	17	1,317	1,313	1,306	1,112	1,55
18	9	18	1,45	1,442	1,429	1,192	1,777
18	12	1	0,625	0,618	0,622	0,422	0,787
18	12	2	0,706	0,702	0,704	0,55	0,844
18	12	3	0,758	0,755	0,754	0,62	0,887
18	12	4	0,8	0,798	0,797	0,668	0,924
18	12	5	0,836	0,834	0,834	0,709	0,956
18	12	6	0,869	0,867	0,867	0,747	0,99
18	12	7	0,9	0,898	0,898	0,78	1,019
18	12	8	0,929	0,927	0,927	0,809	1,048
18	12	9	0,959	0,957	0,956	0,838	1,079
18	12	10	0,988	0,986	0,985	0,865	1,109
18	12	11	1,017	1,015	1,015	0,893	1,141
18	12	12	1,048	1,046	1,045	0,923	1,174
18	12	13	1,081	1,079	1,078	0,953	1,212
18	12	14	1,117	1,115	1,113	0,984	1,259
18	12	15	1,159	1,156	1,153	1,017	1,308
18	12	16	1,208	1,205	1,202	1,056	1,37
18	12	17	1,273	1,27	1,265	1,101	1,464
18	12	18	1,385	1,379	1,37	1,168	1,652
18	16	1	0,675	0,67	0,674	0,495	0,82
18	16	2	0,747	0,744	0,745	0,604	0,87
18	16	3	0,793	0,791	0,793	0,67	0,908
18	16	4	0,83	0,828	0,83	0,712	0,939
18	16	5	0,862	0,86	0,861	0,75	0,968
18	16	6	0,89	0,888	0,889	0,785	0,993
18	16	7	0,917	0,915	0,915	0,812	1,019
18	16	8	0,942	0,941	0,94	0,839	1,044
18	16	9	0,967	0,966	0,966	0,865	1,067
18	16	10	0,992	0,991	0,99	0,888	1,096
18	16	11	1,018	1,016	1,016	0,911	1,123
18	16	12	1,044	1,043	1,043	0,936	1,154
18	16	13	1,072	1,071	1,07	0,961	1,186
18	16	14	1,104	1,102	1,101	0,989	1,224
18	16	15	1,139	1,137	1,135	1,018	1,266
18	16	16	1,18	1,178	1,175	1,051	1,321
18	16	17	1,237	1,234	1,229	1,09	1,406

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
18	16	18	1,331	1,327	1,317	1,144	1,565
18	20	1	0,71	0,706	0,71	0,547	0,84
18	20	2	0,776	0,774	0,776	0,651	0,885
18	20	3	0,818	0,816	0,816	0,709	0,918
18	20	4	0,85	0,849	0,849	0,75	0,947
18	20	5	0,878	0,877	0,877	0,78	0,974
18	20	6	0,903	0,902	0,902	0,807	0,997
18	20	7	0,927	0,926	0,926	0,834	1,02
18	20	8	0,95	0,949	0,948	0,857	1,042
18	20	9	0,972	0,97	0,97	0,879	1,062
18	20	10	0,994	0,993	0,993	0,901	1,088
18	20	11	1,017	1,016	1,016	0,923	1,113
18	20	12	1,041	1,04	1,039	0,945	1,139
18	20	13	1,066	1,065	1,063	0,967	1,168
18	20	14	1,094	1,092	1,091	0,992	1,2
18	20	15	1,125	1,123	1,121	1,019	1,239
18	20	16	1,162	1,16	1,158	1,047	1,287
18	20	17	1,212	1,209	1,205	1,082	1,362
18	20	18	1,294	1,291	1,283	1,132	1,499
19	2	1	0,088	0,063	0,045	0,002	0,224
19	2	2	0,152	0,127	0,11	0,017	0,335
19	2	3	0,216	0,191	0,175	0,043	0,43
19	2	4	0,281	0,256	0,241	0,077	0,522
19	2	5	0,346	0,322	0,308	0,115	0,607
19	2	6	0,414	0,39	0,376	0,158	0,695
19	2	7	0,483	0,459	0,447	0,206	0,782
19	2	8	0,554	0,531	0,52	0,257	0,871
19	2	9	0,629	0,607	0,595	0,313	0,963
19	2	10	0,708	0,685	0,675	0,372	1,063
19	2	11	0,791	0,769	0,758	0,433	1,164
19	2	12	0,881	0,858	0,848	0,502	1,272
19	2	13	0,978	0,955	0,944	0,576	1,398
19	2	14	1,086	1,063	1,052	0,657	1,531
19	2	15	1,208	1,184	1,172	0,747	1,689
19	2	16	1,351	1,325	1,31	0,853	1,875
19	2	17	1,527	1,499	1,482	0,973	2,125
19	2	18	1,77	1,736	1,713	1,127	2,479
19	2	19	2,194	2,143	2,097	1,349	3,198
19	3	1	0,23	0,203	0,192	0,036	0,44
19	3	2	0,329	0,309	0,3	0,114	0,547
19	3	3	0,408	0,392	0,386	0,184	0,634
19	3	4	0,479	0,464	0,459	0,249	0,708
19	3	5	0,544	0,53	0,526	0,309	0,78
19	3	6	0,606	0,593	0,588	0,366	0,844
19	3	7	0,666	0,654	0,649	0,42	0,912

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
19	3	8	0,726	0,714	0,71	0,477	0,978
19	3	9	0,785	0,774	0,77	0,529	1,044
19	3	10	0,846	0,835	0,831	0,583	1,11
19	3	11	0,909	0,898	0,895	0,637	1,182
19	3	12	0,975	0,964	0,96	0,694	1,26
19	3	13	1,046	1,035	1,03	0,752	1,342
19	3	14	1,123	1,111	1,107	0,816	1,435
19	3	15	1,209	1,197	1,191	0,887	1,54
19	3	16	1,308	1,295	1,287	0,962	1,671
19	3	17	1,43	1,416	1,406	1,052	1,836
19	3	18	1,596	1,578	1,563	1,163	2,086
19	3	19	1,883	1,856	1,824	1,318	2,573
19	6	1	0,456	0,442	0,443	0,216	0,659
19	6	2	0,556	0,547	0,547	0,353	0,741
19	6	3	0,625	0,618	0,617	0,437	0,797
19	6	4	0,681	0,675	0,674	0,504	0,851
19	6	5	0,73	0,724	0,723	0,558	0,895
19	6	6	0,774	0,77	0,768	0,605	0,94
19	6	7	0,817	0,813	0,812	0,648	0,984
19	6	8	0,858	0,854	0,853	0,687	1,025
19	6	9	0,898	0,894	0,893	0,727	1,067
19	6	10	0,939	0,934	0,933	0,767	1,111
19	6	11	0,98	0,975	0,974	0,805	1,157
19	6	12	1,022	1,018	1,016	0,844	1,204
19	6	13	1,067	1,062	1,061	0,884	1,256
19	6	14	1,114	1,11	1,108	0,925	1,311
19	6	15	1,168	1,163	1,16	0,97	1,374
19	6	16	1,228	1,223	1,22	1,017	1,449
19	6	17	1,303	1,297	1,292	1,073	1,55
19	6	18	1,402	1,395	1,387	1,142	1,697
19	6	19	1,574	1,563	1,544	1,24	1,993
19	9	1	0,558	0,549	0,553	0,342	0,737
19	9	2	0,648	0,642	0,644	0,472	0,803
19	9	3	0,706	0,702	0,702	0,546	0,85
19	9	4	0,753	0,749	0,749	0,604	0,892
19	9	5	0,793	0,79	0,79	0,65	0,93
19	9	6	0,83	0,827	0,827	0,69	0,966
19	9	7	0,865	0,862	0,861	0,727	1,001
19	9	8	0,898	0,895	0,895	0,761	1,033
19	9	9	0,93	0,927	0,926	0,793	1,066
19	9	10	0,962	0,959	0,958	0,825	1,099
19	9	11	0,995	0,992	0,991	0,856	1,134
19	9	12	1,028	1,026	1,025	0,887	1,172
19	9	13	1,064	1,061	1,06	0,919	1,211
19	9	14	1,102	1,099	1,097	0,951	1,255

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
19	9	15	1,143	1,141	1,138	0,986	1,306
19	9	16	1,191	1,188	1,185	1,024	1,366
19	9	17	1,248	1,245	1,241	1,068	1,443
19	9	18	1,326	1,321	1,315	1,123	1,553
19	9	19	1,459	1,451	1,436	1,201	1,781
19	12	1	0,619	0,612	0,616	0,419	0,781
19	12	2	0,7	0,696	0,698	0,546	0,836
19	12	3	0,752	0,749	0,75	0,612	0,876
19	12	4	0,793	0,79	0,79	0,664	0,914
19	12	5	0,828	0,825	0,826	0,702	0,946
19	12	6	0,859	0,857	0,857	0,74	0,975
19	12	7	0,889	0,887	0,886	0,772	1,005
19	12	8	0,917	0,915	0,915	0,801	1,033
19	12	9	0,945	0,943	0,942	0,829	1,062
19	12	10	0,972	0,971	0,97	0,857	1,091
19	12	11	1	0,998	0,997	0,884	1,119
19	12	12	1,029	1,027	1,027	0,908	1,15
19	12	13	1,059	1,057	1,055	0,936	1,184
19	12	14	1,091	1,089	1,088	0,964	1,222
19	12	15	1,127	1,124	1,122	0,993	1,264
19	12	16	1,167	1,165	1,162	1,027	1,318
19	12	17	1,215	1,213	1,21	1,064	1,38
19	12	18	1,281	1,277	1,272	1,109	1,472
19	12	19	1,392	1,386	1,375	1,176	1,663
19	16	1	0,671	0,666	0,67	0,495	0,816
19	16	2	0,743	0,74	0,742	0,602	0,864
19	16	3	0,787	0,785	0,787	0,666	0,898
19	16	4	0,823	0,821	0,822	0,711	0,929
19	16	5	0,854	0,852	0,852	0,746	0,956
19	16	6	0,882	0,88	0,88	0,777	0,982
19	16	7	0,907	0,905	0,906	0,803	1,007
19	16	8	0,931	0,93	0,93	0,828	1,03
19	16	9	0,955	0,954	0,954	0,854	1,054
19	16	10	0,979	0,977	0,977	0,877	1,077
19	16	11	1,003	1,001	1,001	0,9	1,104
19	16	12	1,027	1,026	1,025	0,924	1,131
19	16	13	1,053	1,051	1,051	0,948	1,161
19	16	14	1,08	1,079	1,078	0,973	1,193
19	16	15	1,111	1,109	1,107	0,998	1,228
19	16	16	1,145	1,143	1,141	1,027	1,269
19	16	17	1,186	1,184	1,18	1,057	1,324
19	16	18	1,241	1,239	1,234	1,098	1,406
19	16	19	1,335	1,331	1,321	1,152	1,567
20	2	1	0,083	0,06	0,043	0,002	0,212
20	2	2	0,144	0,12	0,104	0,015	0,317

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
20	2	3	0,205	0,181	0,165	0,041	0,409
20	2	4	0,266	0,243	0,228	0,072	0,494
20	2	5	0,329	0,306	0,292	0,11	0,579
20	2	6	0,392	0,37	0,356	0,15	0,661
20	2	7	0,458	0,435	0,423	0,195	0,745
20	2	8	0,524	0,502	0,49	0,242	0,827
20	2	9	0,595	0,573	0,561	0,293	0,914
20	2	10	0,668	0,646	0,635	0,348	1,003
20	2	11	0,744	0,723	0,713	0,407	1,1
20	2	12	0,826	0,804	0,794	0,469	1,2
20	2	13	0,914	0,892	0,881	0,536	1,307
20	2	14	1,009	0,987	0,977	0,608	1,426
20	2	15	1,115	1,092	1,081	0,689	1,56
20	2	16	1,235	1,211	1,198	0,778	1,715
20	2	17	1,376	1,351	1,337	0,876	1,901
20	2	18	1,55	1,523	1,507	0,996	2,139
20	2	19	1,791	1,757	1,734	1,153	2,499
20	2	20	2,212	2,162	2,116	1,373	3,216
20	3	1	0,224	0,198	0,186	0,035	0,43
20	3	2	0,32	0,301	0,293	0,111	0,535
20	3	3	0,398	0,382	0,376	0,181	0,617
20	3	4	0,466	0,452	0,446	0,243	0,69
20	3	5	0,529	0,516	0,511	0,301	0,759
20	3	6	0,589	0,577	0,572	0,356	0,823
20	3	7	0,647	0,635	0,631	0,41	0,885
20	3	8	0,704	0,693	0,689	0,461	0,947
20	3	9	0,761	0,75	0,746	0,513	1,011
20	3	10	0,819	0,808	0,804	0,565	1,076
20	3	11	0,878	0,868	0,863	0,616	1,144
20	3	12	0,94	0,929	0,925	0,668	1,213
20	3	13	1,004	0,993	0,989	0,725	1,287
20	3	14	1,073	1,062	1,057	0,785	1,368
20	3	15	1,148	1,137	1,131	0,847	1,457
20	3	16	1,232	1,221	1,215	0,912	1,566
20	3	17	1,33	1,317	1,309	0,986	1,695
20	3	18	1,45	1,436	1,426	1,076	1,862
20	3	19	1,613	1,596	1,581	1,183	2,098
20	3	20	1,897	1,871	1,838	1,339	2,592
20	6	1	0,45	0,436	0,438	0,216	0,648
20	6	2	0,549	0,54	0,541	0,348	0,729
20	6	3	0,616	0,61	0,609	0,432	0,789
20	6	4	0,671	0,665	0,665	0,495	0,837
20	6	5	0,718	0,713	0,713	0,548	0,882
20	6	6	0,762	0,757	0,755	0,594	0,924
20	6	7	0,802	0,798	0,797	0,636	0,964

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
20	6	8	0,842	0,837	0,837	0,676	1,004
20	6	9	0,88	0,876	0,874	0,714	1,044
20	6	10	0,918	0,914	0,913	0,753	1,084
20	6	11	0,957	0,953	0,952	0,789	1,126
20	6	12	0,996	0,992	0,991	0,825	1,17
20	6	13	1,038	1,034	1,031	0,862	1,218
20	6	14	1,081	1,077	1,074	0,898	1,268
20	6	15	1,129	1,124	1,121	0,942	1,323
20	6	16	1,181	1,176	1,173	0,984	1,388
20	6	17	1,241	1,236	1,232	1,031	1,464
20	6	18	1,314	1,308	1,302	1,085	1,563
20	6	19	1,412	1,405	1,396	1,153	1,708
20	6	20	1,582	1,571	1,551	1,249	1,997
20	9	1	0,553	0,544	0,548	0,337	0,731
20	9	2	0,64	0,635	0,636	0,469	0,794
20	9	3	0,699	0,694	0,695	0,541	0,844
20	9	4	0,744	0,741	0,741	0,597	0,882
20	9	5	0,784	0,781	0,781	0,641	0,918
20	9	6	0,819	0,816	0,816	0,68	0,951
20	9	7	0,853	0,85	0,849	0,718	0,983
20	9	8	0,884	0,882	0,881	0,752	1,016
20	9	9	0,915	0,913	0,912	0,784	1,046
20	9	10	0,946	0,943	0,942	0,812	1,078
20	9	11	0,977	0,974	0,973	0,841	1,113
20	9	12	1,008	1,005	1,004	0,871	1,145
20	9	13	1,04	1,038	1,037	0,9	1,183
20	9	14	1,074	1,071	1,069	0,933	1,221
20	9	15	1,111	1,109	1,107	0,965	1,263
20	9	16	1,152	1,149	1,147	0,997	1,312
20	9	17	1,198	1,195	1,192	1,036	1,37
20	9	18	1,255	1,252	1,247	1,079	1,451
20	9	19	1,333	1,328	1,321	1,13	1,563
20	9	20	1,465	1,457	1,443	1,207	1,786
20	12	1	0,616	0,609	0,615	0,42	0,773
20	12	2	0,695	0,691	0,692	0,542	0,829
20	12	3	0,746	0,743	0,744	0,608	0,868
20	12	4	0,786	0,783	0,784	0,658	0,903
20	12	5	0,82	0,817	0,818	0,698	0,934
20	12	6	0,85	0,848	0,848	0,736	0,963
20	12	7	0,879	0,877	0,877	0,765	0,99
20	12	8	0,906	0,904	0,904	0,791	1,019
20	12	9	0,933	0,931	0,93	0,818	1,047
20	12	10	0,959	0,957	0,957	0,846	1,071
20	12	11	0,985	0,984	0,984	0,869	1,102
20	12	12	1,012	1,01	1,01	0,897	1,13

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
20	12	13	1,04	1,038	1,037	0,92	1,161
20	12	14	1,069	1,068	1,066	0,947	1,194
20	12	15	1,101	1,099	1,097	0,975	1,232
20	12	16	1,135	1,133	1,131	1,005	1,276
20	12	17	1,175	1,172	1,171	1,037	1,322
20	12	18	1,222	1,22	1,216	1,074	1,387
20	12	19	1,287	1,283	1,277	1,118	1,485
20	12	20	1,397	1,392	1,379	1,181	1,67
20	16	1	0,666	0,661	0,665	0,491	0,81
20	16	2	0,737	0,735	0,737	0,602	0,856
20	16	3	0,782	0,78	0,781	0,661	0,891
20	16	4	0,817	0,815	0,816	0,706	0,922
20	16	5	0,847	0,845	0,845	0,739	0,947
20	16	6	0,873	0,872	0,872	0,77	0,973
20	16	7	0,898	0,897	0,897	0,798	0,996
20	16	8	0,922	0,92	0,92	0,823	1,019
20	16	9	0,945	0,944	0,943	0,847	1,044
20	16	10	0,968	0,966	0,966	0,868	1,065
20	16	11	0,99	0,988	0,988	0,889	1,089
20	16	12	1,012	1,011	1,01	0,911	1,112
20	16	13	1,036	1,035	1,034	0,933	1,138
20	16	14	1,061	1,06	1,059	0,957	1,166
20	16	15	1,088	1,087	1,086	0,979	1,201
20	16	16	1,117	1,116	1,114	1,007	1,234
20	16	17	1,151	1,149	1,147	1,034	1,275
20	16	18	1,192	1,19	1,187	1,065	1,327
20	16	19	1,246	1,243	1,239	1,104	1,41
20	16	20	1,34	1,336	1,324	1,159	1,578
22	2	1	0,076	0,055	0,039	0,001	0,196
22	2	2	0,132	0,11	0,095	0,014	0,29
22	2	3	0,188	0,166	0,152	0,037	0,375
22	2	4	0,243	0,221	0,208	0,065	0,454
22	2	5	0,3	0,278	0,266	0,098	0,529
22	2	6	0,357	0,336	0,324	0,135	0,605
22	2	7	0,416	0,395	0,384	0,175	0,678
22	2	8	0,476	0,456	0,446	0,218	0,752
22	2	9	0,538	0,518	0,509	0,264	0,829
22	2	10	0,602	0,583	0,574	0,311	0,909
22	2	11	0,67	0,65	0,641	0,363	0,992
22	2	12	0,74	0,72	0,711	0,417	1,076
22	2	13	0,814	0,794	0,785	0,475	1,164
22	2	14	0,892	0,873	0,864	0,536	1,26
22	2	15	0,977	0,957	0,947	0,602	1,367
22	2	16	1,069	1,049	1,038	0,674	1,482
22	2	17	1,172	1,151	1,14	0,755	1,612

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
22	2	18	1,289	1,268	1,256	0,84	1,762
22	2	19	1,427	1,404	1,389	0,94	1,944
22	2	20	1,598	1,572	1,556	1,059	2,179
22	2	21	1,835	1,803	1,781	1,204	2,536
22	2	22	2,253	2,204	2,157	1,426	3,255
22	3	1	0,213	0,189	0,178	0,035	0,409
22	3	2	0,305	0,287	0,28	0,106	0,508
22	3	3	0,378	0,363	0,358	0,172	0,585
22	3	4	0,443	0,429	0,424	0,23	0,655
22	3	5	0,501	0,489	0,485	0,285	0,718
22	3	6	0,557	0,545	0,541	0,336	0,778
22	3	7	0,61	0,599	0,595	0,387	0,836
22	3	8	0,663	0,652	0,649	0,434	0,892
22	3	9	0,715	0,705	0,702	0,481	0,947
22	3	10	0,767	0,757	0,754	0,529	1,006
22	3	11	0,819	0,81	0,806	0,577	1,065
22	3	12	0,873	0,864	0,859	0,625	1,125
22	3	13	0,929	0,92	0,915	0,674	1,188
22	3	14	0,987	0,978	0,974	0,723	1,253
22	3	15	1,049	1,039	1,034	0,775	1,328
22	3	16	1,115	1,105	1,101	0,83	1,405
22	3	17	1,188	1,178	1,173	0,89	1,494
22	3	18	1,27	1,259	1,253	0,954	1,599
22	3	19	1,364	1,353	1,345	1,026	1,723
22	3	20	1,481	1,468	1,459	1,112	1,883
22	3	21	1,642	1,626	1,611	1,214	2,123
22	3	22	1,922	1,896	1,864	1,369	2,604
22	6	1	0,442	0,428	0,429	0,21	0,637
22	6	2	0,537	0,528	0,529	0,343	0,712
22	6	3	0,602	0,595	0,594	0,423	0,768
22	6	4	0,654	0,648	0,648	0,484	0,816
22	6	5	0,7	0,695	0,694	0,533	0,858
22	6	6	0,741	0,736	0,735	0,578	0,896
22	6	7	0,779	0,775	0,774	0,618	0,938
22	6	8	0,816	0,812	0,81	0,657	0,972
22	6	9	0,851	0,847	0,846	0,693	1,008
22	6	10	0,886	0,882	0,881	0,725	1,045
22	6	11	0,921	0,917	0,916	0,761	1,082
22	6	12	0,956	0,952	0,952	0,794	1,12
22	6	13	0,991	0,988	0,987	0,827	1,159
22	6	14	1,028	1,025	1,023	0,86	1,198
22	6	15	1,067	1,064	1,061	0,893	1,24
22	6	16	1,109	1,105	1,102	0,929	1,29
22	6	17	1,154	1,15	1,147	0,967	1,347
22	6	18	1,204	1,2	1,197	1,009	1,406

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
22	6	19	1,262	1,258	1,255	1,056	1,483
22	6	20	1,334	1,329	1,323	1,11	1,579
22	6	21	1,43	1,424	1,414	1,176	1,723
22	6	22	1,597	1,586	1,567	1,262	2,011
22	9	1	0,545	0,536	0,541	0,336	0,719
22	9	2	0,63	0,625	0,626	0,463	0,776
22	9	3	0,685	0,681	0,681	0,535	0,823
22	9	4	0,73	0,726	0,727	0,588	0,863
22	9	5	0,767	0,764	0,765	0,631	0,897
22	9	6	0,802	0,799	0,799	0,671	0,929
22	9	7	0,833	0,83	0,83	0,704	0,96
22	9	8	0,863	0,86	0,86	0,732	0,989
22	9	9	0,892	0,889	0,889	0,762	1,018
22	9	10	0,92	0,918	0,918	0,79	1,046
22	9	11	0,948	0,946	0,945	0,817	1,074
22	9	12	0,976	0,974	0,973	0,845	1,104
22	9	13	1,005	1,002	1,001	0,871	1,138
22	9	14	1,034	1,032	1,03	0,899	1,171
22	9	15	1,064	1,062	1,061	0,927	1,204
22	9	16	1,097	1,094	1,091	0,957	1,242
22	9	17	1,132	1,13	1,127	0,987	1,284
22	9	18	1,172	1,169	1,166	1,018	1,334
22	9	19	1,217	1,214	1,21	1,055	1,391
22	9	20	1,273	1,269	1,265	1,096	1,465
22	9	21	1,348	1,343	1,337	1,154	1,57
22	9	22	1,476	1,469	1,456	1,226	1,788
22	12	1	0,608	0,601	0,607	0,412	0,762
22	12	2	0,685	0,681	0,682	0,539	0,817
22	12	3	0,733	0,73	0,731	0,604	0,856
22	12	4	0,772	0,77	0,77	0,649	0,888
22	12	5	0,805	0,803	0,802	0,689	0,917
22	12	6	0,834	0,833	0,832	0,719	0,945
22	12	7	0,862	0,86	0,859	0,75	0,969
22	12	8	0,887	0,886	0,885	0,777	0,995
22	12	9	0,912	0,91	0,909	0,803	1,019
22	12	10	0,936	0,934	0,933	0,827	1,046
22	12	11	0,959	0,958	0,957	0,85	1,071
22	12	12	0,983	0,982	0,981	0,874	1,096
22	12	13	1,008	1,006	1,005	0,897	1,121
22	12	14	1,033	1,031	1,03	0,921	1,147
22	12	15	1,059	1,057	1,055	0,943	1,176
22	12	16	1,087	1,085	1,083	0,968	1,209
22	12	17	1,116	1,114	1,112	0,995	1,245
22	12	18	1,149	1,147	1,144	1,025	1,288
22	12	19	1,188	1,186	1,183	1,056	1,334

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
22	12	20	1,234	1,232	1,228	1,09	1,393
22	12	21	1,297	1,294	1,288	1,134	1,487
22	12	22	1,406	1,4	1,39	1,194	1,672
22	16	1	0,659	0,654	0,659	0,486	0,797
22	16	2	0,728	0,725	0,727	0,593	0,845
22	16	3	0,772	0,77	0,771	0,655	0,879
22	16	4	0,807	0,805	0,805	0,698	0,908
22	16	5	0,835	0,834	0,835	0,733	0,933
22	16	6	0,861	0,859	0,86	0,76	0,956
22	16	7	0,884	0,883	0,883	0,788	0,979
22	16	8	0,906	0,905	0,905	0,811	0,999
22	16	9	0,928	0,927	0,927	0,834	1,02
22	16	10	0,949	0,947	0,947	0,856	1,04
22	16	11	0,969	0,968	0,968	0,876	1,061
22	16	12	0,99	0,989	0,989	0,895	1,084
22	16	13	1,011	1,01	1,01	0,915	1,105
22	16	14	1,032	1,031	1,03	0,935	1,13
22	16	15	1,054	1,053	1,052	0,956	1,154
22	16	16	1,077	1,076	1,075	0,975	1,182
22	16	17	1,103	1,102	1,101	1	1,211
22	16	18	1,132	1,13	1,129	1,024	1,245
22	16	19	1,163	1,162	1,16	1,048	1,285
22	16	20	1,203	1,201	1,199	1,079	1,342
22	16	21	1,256	1,254	1,25	1,113	1,414
22	16	22	1,347	1,343	1,335	1,168	1,571
24	2	1	0,07	0,05	0,036	0,001	0,18
24	2	2	0,121	0,101	0,086	0,013	0,268
24	2	3	0,172	0,152	0,138	0,034	0,346
24	2	4	0,223	0,203	0,19	0,06	0,418
24	2	5	0,275	0,255	0,243	0,09	0,489
24	2	6	0,327	0,308	0,296	0,123	0,558
24	2	7	0,38	0,361	0,35	0,158	0,624
24	2	8	0,435	0,416	0,406	0,196	0,689
24	2	9	0,491	0,472	0,463	0,238	0,758
24	2	10	0,548	0,53	0,521	0,281	0,829
24	2	11	0,608	0,589	0,581	0,325	0,9
24	2	12	0,669	0,651	0,643	0,373	0,976
24	2	13	0,733	0,715	0,706	0,422	1,055
24	2	14	0,8	0,782	0,774	0,475	1,132
24	2	15	0,872	0,854	0,846	0,531	1,221
24	2	16	0,948	0,93	0,922	0,594	1,314
24	2	17	1,03	1,012	1,003	0,659	1,414
24	2	18	1,12	1,101	1,091	0,727	1,526
24	2	19	1,22	1,2	1,191	0,803	1,651
24	2	20	1,334	1,314	1,303	0,889	1,8

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
24	2	21	1,469	1,447	1,435	0,987	1,982
24	2	22	1,637	1,612	1,596	1,103	2,209
24	2	23	1,871	1,841	1,819	1,249	2,567
24	2	24	2,284	2,236	2,189	1,465	3,272
24	3	1	0,204	0,181	0,17	0,032	0,391
24	3	2	0,292	0,274	0,268	0,102	0,487
24	3	3	0,361	0,347	0,341	0,163	0,563
24	3	4	0,422	0,409	0,405	0,22	0,628
24	3	5	0,478	0,466	0,462	0,272	0,685
24	3	6	0,53	0,519	0,515	0,322	0,739
24	3	7	0,58	0,57	0,566	0,369	0,792
24	3	8	0,629	0,619	0,615	0,413	0,846
24	3	9	0,677	0,668	0,663	0,458	0,899
24	3	10	0,725	0,716	0,712	0,502	0,951
24	3	11	0,772	0,763	0,76	0,545	1,001
24	3	12	0,82	0,812	0,808	0,588	1,055
24	3	13	0,87	0,861	0,858	0,633	1,11
24	3	14	0,92	0,912	0,908	0,676	1,167
24	3	15	0,973	0,965	0,961	0,721	1,229
24	3	16	1,029	1,02	1,016	0,769	1,294
24	3	17	1,088	1,079	1,075	0,82	1,364
24	3	18	1,151	1,142	1,137	0,873	1,441
24	3	19	1,222	1,212	1,207	0,929	1,527
24	3	20	1,302	1,292	1,286	0,991	1,629
24	3	21	1,395	1,384	1,377	1,06	1,749
24	3	22	1,51	1,497	1,488	1,145	1,907
24	3	23	1,668	1,652	1,637	1,246	2,142
24	3	24	1,943	1,918	1,887	1,395	2,615
24	6	1	0,432	0,419	0,42	0,21	0,621
24	6	2	0,525	0,517	0,517	0,338	0,691
24	6	3	0,587	0,581	0,581	0,414	0,744
24	6	4	0,638	0,633	0,633	0,472	0,792
24	6	5	0,681	0,677	0,677	0,522	0,831
24	6	6	0,721	0,716	0,715	0,563	0,869
24	6	7	0,757	0,753	0,753	0,604	0,906
24	6	8	0,791	0,788	0,786	0,64	0,943
24	6	9	0,825	0,821	0,821	0,673	0,976
24	6	10	0,857	0,854	0,853	0,705	1,007
24	6	11	0,889	0,886	0,885	0,736	1,039
24	6	12	0,921	0,918	0,917	0,767	1,073
24	6	13	0,954	0,95	0,949	0,797	1,11
24	6	14	0,986	0,983	0,982	0,829	1,146
24	6	15	1,02	1,017	1,016	0,86	1,183
24	6	16	1,055	1,051	1,049	0,891	1,221
24	6	17	1,092	1,089	1,086	0,924	1,261

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
24	6	18	1,132	1,129	1,127	0,959	1,311
24	6	19	1,175	1,172	1,169	0,996	1,361
24	6	20	1,224	1,22	1,217	1,034	1,422
24	6	21	1,28	1,276	1,272	1,079	1,497
24	6	22	1,349	1,344	1,339	1,129	1,593
24	6	23	1,444	1,438	1,429	1,195	1,729
24	6	24	1,61	1,599	1,581	1,284	2,006
24	9	1	0,538	0,53	0,534	0,334	0,705
24	9	2	0,622	0,617	0,618	0,458	0,768
24	9	3	0,676	0,672	0,673	0,53	0,811
24	9	4	0,718	0,714	0,714	0,579	0,847
24	9	5	0,754	0,751	0,751	0,621	0,88
24	9	6	0,786	0,783	0,782	0,658	0,909
24	9	7	0,816	0,813	0,813	0,691	0,94
24	9	8	0,844	0,841	0,841	0,721	0,967
24	9	9	0,87	0,868	0,868	0,749	0,99
24	9	10	0,897	0,894	0,894	0,774	1,017
24	9	11	0,923	0,921	0,919	0,8	1,044
24	9	12	0,948	0,946	0,945	0,824	1,071
24	9	13	0,974	0,972	0,971	0,85	1,096
24	9	14	1	0,998	0,998	0,875	1,125
24	9	15	1,027	1,024	1,024	0,899	1,155
24	9	16	1,054	1,052	1,051	0,926	1,185
24	9	17	1,084	1,082	1,08	0,952	1,221
24	9	18	1,115	1,113	1,111	0,98	1,253
24	9	19	1,149	1,147	1,145	1,011	1,294
24	9	20	1,187	1,185	1,182	1,042	1,343
24	9	21	1,231	1,228	1,225	1,072	1,395
24	9	22	1,285	1,282	1,278	1,116	1,471
24	9	23	1,359	1,355	1,349	1,167	1,577
24	9	24	1,489	1,482	1,467	1,239	1,806
24	12	1	0,601	0,594	0,599	0,409	0,754
24	12	2	0,675	0,671	0,674	0,527	0,805
24	12	3	0,723	0,721	0,721	0,593	0,843
24	12	4	0,761	0,759	0,759	0,637	0,873
24	12	5	0,793	0,791	0,791	0,678	0,902
24	12	6	0,821	0,819	0,819	0,71	0,929
24	12	7	0,847	0,845	0,845	0,739	0,953
24	12	8	0,871	0,87	0,869	0,764	0,977
24	12	9	0,895	0,893	0,892	0,789	1,001
24	12	10	0,917	0,915	0,915	0,812	1,022
24	12	11	0,939	0,938	0,937	0,834	1,045
24	12	12	0,961	0,959	0,959	0,854	1,066
24	12	13	0,983	0,981	0,981	0,876	1,09
24	12	14	1,005	1,003	1,003	0,897	1,111

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
24	12	15	1,027	1,026	1,025	0,917	1,134
24	12	16	1,051	1,05	1,049	0,94	1,162
24	12	17	1,076	1,075	1,074	0,96	1,189
24	12	18	1,103	1,101	1,1	0,986	1,221
24	12	19	1,132	1,13	1,129	1,012	1,255
24	12	20	1,164	1,162	1,161	1,039	1,294
24	12	21	1,201	1,199	1,196	1,071	1,342
24	12	22	1,246	1,244	1,241	1,103	1,404
24	12	23	1,309	1,306	1,301	1,145	1,498
24	12	24	1,417	1,412	1,401	1,205	1,685
26	2	1	0,065	0,047	0,033	0,001	0,166
26	2	2	0,113	0,094	0,08	0,012	0,249
26	2	3	0,16	0,141	0,128	0,031	0,322
26	2	4	0,207	0,188	0,176	0,055	0,39
26	2	5	0,255	0,236	0,225	0,083	0,453
26	2	6	0,303	0,285	0,274	0,113	0,519
26	2	7	0,352	0,334	0,324	0,146	0,58
26	2	8	0,402	0,384	0,374	0,181	0,645
26	2	9	0,453	0,435	0,425	0,218	0,706
26	2	10	0,505	0,487	0,478	0,256	0,77
26	2	11	0,558	0,541	0,532	0,297	0,834
26	2	12	0,614	0,597	0,588	0,34	0,9
26	2	13	0,671	0,654	0,646	0,384	0,969
26	2	14	0,73	0,713	0,705	0,432	1,041
26	2	15	0,793	0,776	0,768	0,481	1,116
26	2	16	0,859	0,842	0,833	0,533	1,195
26	2	17	0,928	0,911	0,902	0,59	1,279
26	2	18	1,002	0,985	0,977	0,649	1,367
26	2	19	1,082	1,064	1,056	0,709	1,467
26	2	20	1,169	1,151	1,144	0,777	1,576
26	2	21	1,268	1,249	1,24	0,852	1,703
26	2	22	1,379	1,36	1,349	0,936	1,842
26	2	23	1,512	1,491	1,479	1,032	2,019
26	2	24	1,677	1,653	1,637	1,145	2,251
26	2	25	1,907	1,877	1,852	1,29	2,596
26	2	26	2,316	2,27	2,224	1,502	3,296
26	3	1	0,196	0,174	0,163	0,032	0,377
26	3	2	0,28	0,263	0,256	0,097	0,469
26	3	3	0,346	0,332	0,327	0,156	0,537
26	3	4	0,404	0,392	0,387	0,211	0,598
26	3	5	0,456	0,445	0,442	0,26	0,653
26	3	6	0,506	0,496	0,492	0,306	0,707
26	3	7	0,554	0,544	0,541	0,349	0,758
26	3	8	0,6	0,59	0,587	0,394	0,807
26	3	9	0,644	0,635	0,633	0,434	0,853

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
26	3	10	0,689	0,68	0,677	0,472	0,9
26	3	11	0,733	0,724	0,722	0,515	0,948
26	3	12	0,776	0,768	0,765	0,555	0,996
26	3	13	0,821	0,813	0,81	0,596	1,044
26	3	14	0,867	0,859	0,856	0,637	1,097
26	3	15	0,913	0,905	0,903	0,68	1,151
26	3	16	0,962	0,954	0,951	0,72	1,206
26	3	17	1,013	1,005	1,001	0,766	1,264
26	3	18	1,066	1,058	1,053	0,81	1,323
26	3	19	1,123	1,115	1,111	0,86	1,393
26	3	20	1,185	1,176	1,172	0,909	1,466
26	3	21	1,253	1,244	1,241	0,966	1,549
26	3	22	1,331	1,321	1,317	1,026	1,647
26	3	23	1,422	1,412	1,405	1,094	1,77
26	3	24	1,535	1,523	1,513	1,175	1,927
26	3	25	1,691	1,676	1,66	1,275	2,161
26	3	26	1,966	1,941	1,907	1,421	2,641
26	6	1	0,423	0,41	0,411	0,201	0,609
26	6	2	0,514	0,506	0,507	0,328	0,681
26	6	3	0,575	0,569	0,569	0,408	0,73
26	6	4	0,624	0,619	0,618	0,466	0,774
26	6	5	0,666	0,662	0,661	0,513	0,813
26	6	6	0,703	0,699	0,699	0,552	0,849
26	6	7	0,738	0,734	0,734	0,589	0,884
26	6	8	0,771	0,768	0,767	0,625	0,914
26	6	9	0,802	0,799	0,798	0,656	0,947
26	6	10	0,833	0,829	0,829	0,688	0,978
26	6	11	0,863	0,859	0,859	0,717	1,007
26	6	12	0,892	0,889	0,888	0,745	1,037
26	6	13	0,922	0,919	0,919	0,773	1,067
26	6	14	0,951	0,948	0,948	0,803	1,098
26	6	15	0,982	0,979	0,978	0,829	1,133
26	6	16	1,012	1,009	1,009	0,859	1,166
26	6	17	1,045	1,042	1,041	0,887	1,202
26	6	18	1,079	1,076	1,074	0,919	1,242
26	6	19	1,115	1,112	1,11	0,949	1,283
26	6	20	1,153	1,15	1,148	0,983	1,326
26	6	21	1,196	1,192	1,189	1,019	1,38
26	6	22	1,243	1,24	1,236	1,059	1,439
26	6	23	1,299	1,295	1,29	1,102	1,509
26	6	24	1,367	1,362	1,357	1,152	1,602
26	6	25	1,46	1,453	1,444	1,212	1,74
26	6	26	1,623	1,613	1,594	1,307	2,025
26	9	1	0,532	0,524	0,526	0,327	0,699
26	9	2	0,614	0,609	0,61	0,459	0,757

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
26	9	3	0,666	0,662	0,662	0,521	0,8
26	9	4	0,707	0,704	0,704	0,573	0,834
26	9	5	0,742	0,739	0,739	0,614	0,864
26	9	6	0,773	0,771	0,771	0,65	0,89
26	9	7	0,802	0,799	0,8	0,68	0,919
26	9	8	0,829	0,827	0,827	0,71	0,945
26	9	9	0,854	0,852	0,852	0,736	0,969
26	9	10	0,879	0,877	0,876	0,762	0,995
26	9	11	0,903	0,901	0,9	0,786	1,018
26	9	12	0,926	0,924	0,924	0,81	1,043
26	9	13	0,949	0,947	0,947	0,831	1,067
26	9	14	0,973	0,971	0,971	0,855	1,093
26	9	15	0,997	0,996	0,994	0,877	1,119
26	9	16	1,022	1,02	1,018	0,9	1,144
26	9	17	1,047	1,045	1,044	0,922	1,171
26	9	18	1,074	1,072	1,071	0,947	1,202
26	9	19	1,102	1,1	1,099	0,973	1,235
26	9	20	1,132	1,13	1,129	0,998	1,269
26	9	21	1,165	1,163	1,162	1,028	1,308
26	9	22	1,202	1,2	1,198	1,057	1,353
26	9	23	1,245	1,243	1,24	1,09	1,409
26	9	24	1,298	1,295	1,292	1,132	1,482
26	9	25	1,369	1,365	1,359	1,178	1,591
26	9	26	1,494	1,487	1,474	1,247	1,8
26	12	1	0,593	0,587	0,59	0,411	0,737
26	12	2	0,667	0,663	0,666	0,528	0,79
26	12	3	0,714	0,711	0,713	0,587	0,828
26	12	4	0,75	0,748	0,749	0,633	0,859
26	12	5	0,781	0,779	0,78	0,665	0,883
26	12	6	0,808	0,806	0,806	0,698	0,909
26	12	7	0,833	0,831	0,832	0,727	0,932
26	12	8	0,856	0,855	0,855	0,75	0,956
26	12	9	0,878	0,877	0,877	0,775	0,978
26	12	10	0,899	0,898	0,898	0,797	0,999
26	12	11	0,92	0,919	0,918	0,819	1,02
26	12	12	0,941	0,939	0,938	0,841	1,042
26	12	13	0,961	0,959	0,958	0,858	1,063
26	12	14	0,981	0,979	0,978	0,88	1,083
26	12	15	1,002	1	0,999	0,899	1,107
26	12	16	1,023	1,021	1,021	0,921	1,127
26	12	17	1,044	1,043	1,041	0,941	1,151
26	12	18	1,067	1,065	1,065	0,96	1,175
26	12	19	1,091	1,089	1,088	0,982	1,204
26	12	20	1,116	1,114	1,113	1,005	1,234
26	12	21	1,144	1,142	1,14	1,028	1,266

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
26	12	22	1,175	1,173	1,171	1,055	1,304
26	12	23	1,211	1,209	1,206	1,084	1,353
26	12	24	1,257	1,254	1,251	1,116	1,417
26	12	25	1,318	1,315	1,309	1,157	1,506
26	12	26	1,425	1,419	1,407	1,216	1,692
28	2	1	0,06	0,043	0,031	0,001	0,154
28	2	2	0,105	0,087	0,074	0,011	0,232
28	2	3	0,148	0,13	0,119	0,028	0,3
28	2	4	0,192	0,174	0,163	0,051	0,361
28	2	5	0,236	0,219	0,208	0,076	0,42
28	2	6	0,281	0,263	0,253	0,103	0,479
28	2	7	0,326	0,309	0,299	0,133	0,537
28	2	8	0,372	0,355	0,346	0,166	0,596
28	2	9	0,419	0,402	0,394	0,2	0,653
28	2	10	0,466	0,45	0,441	0,235	0,712
28	2	11	0,515	0,498	0,49	0,271	0,77
28	2	12	0,565	0,549	0,54	0,31	0,832
28	2	13	0,616	0,6	0,593	0,35	0,891
28	2	14	0,67	0,654	0,646	0,392	0,955
28	2	15	0,725	0,709	0,703	0,437	1,024
28	2	16	0,783	0,767	0,761	0,482	1,092
28	2	17	0,843	0,827	0,82	0,53	1,164
28	2	18	0,906	0,891	0,883	0,583	1,241
28	2	19	0,974	0,958	0,95	0,637	1,321
28	2	20	1,047	1,03	1,022	0,694	1,411
28	2	21	1,125	1,109	1,1	0,757	1,505
28	2	22	1,211	1,194	1,186	0,824	1,612
28	2	23	1,307	1,29	1,28	0,896	1,733
28	2	24	1,417	1,398	1,388	0,978	1,875
28	2	25	1,547	1,527	1,515	1,073	2,05
28	2	26	1,71	1,687	1,672	1,185	2,278
28	2	27	1,937	1,909	1,885	1,329	2,623
28	2	28	2,34	2,296	2,25	1,538	3,322
28	3	1	0,189	0,167	0,157	0,03	0,364
28	3	2	0,27	0,254	0,247	0,094	0,453
28	3	3	0,334	0,32	0,315	0,151	0,519
28	3	4	0,39	0,378	0,374	0,202	0,577
28	3	5	0,439	0,429	0,425	0,249	0,629
28	3	6	0,487	0,477	0,474	0,295	0,678
28	3	7	0,531	0,522	0,519	0,339	0,726
28	3	8	0,575	0,566	0,563	0,379	0,77
28	3	9	0,616	0,608	0,605	0,417	0,815
28	3	10	0,658	0,649	0,646	0,455	0,86
28	3	11	0,698	0,69	0,688	0,493	0,903
28	3	12	0,739	0,732	0,729	0,529	0,948

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
28	3	13	0,78	0,773	0,77	0,568	0,992
28	3	14	0,822	0,815	0,812	0,605	1,037
28	3	15	0,864	0,857	0,855	0,643	1,085
28	3	16	0,908	0,9	0,897	0,683	1,132
28	3	17	0,952	0,945	0,942	0,723	1,183
28	3	18	0,999	0,992	0,989	0,763	1,239
28	3	19	1,048	1,041	1,038	0,805	1,295
28	3	20	1,1	1,092	1,088	0,85	1,356
28	3	21	1,155	1,148	1,144	0,897	1,418
28	3	22	1,216	1,208	1,204	0,948	1,492
28	3	23	1,283	1,275	1,27	1,004	1,577
28	3	24	1,359	1,35	1,344	1,065	1,673
28	3	25	1,448	1,438	1,431	1,13	1,788
28	3	26	1,559	1,548	1,538	1,21	1,944
28	3	27	1,712	1,698	1,683	1,305	2,17
28	3	28	1,982	1,958	1,927	1,447	2,647
28	6	1	0,418	0,405	0,406	0,2	0,6
28	6	2	0,506	0,498	0,499	0,322	0,667
28	6	3	0,565	0,559	0,56	0,398	0,714
28	6	4	0,612	0,607	0,608	0,453	0,757
28	6	5	0,653	0,648	0,649	0,5	0,794
28	6	6	0,689	0,685	0,685	0,541	0,827
28	6	7	0,722	0,719	0,719	0,579	0,86
28	6	8	0,754	0,75	0,75	0,612	0,892
28	6	9	0,784	0,78	0,78	0,642	0,922
28	6	10	0,812	0,809	0,809	0,67	0,951
28	6	11	0,84	0,837	0,837	0,699	0,979
28	6	12	0,868	0,865	0,864	0,726	1,009
28	6	13	0,895	0,892	0,892	0,753	1,036
28	6	14	0,923	0,92	0,919	0,78	1,066
28	6	15	0,951	0,948	0,947	0,807	1,095
28	6	16	0,978	0,976	0,974	0,833	1,122
28	6	17	1,007	1,005	1,003	0,861	1,156
28	6	18	1,037	1,034	1,033	0,888	1,188
28	6	19	1,068	1,065	1,064	0,915	1,224
28	6	20	1,1	1,097	1,096	0,945	1,262
28	6	21	1,135	1,132	1,13	0,976	1,301
28	6	22	1,173	1,169	1,167	1,007	1,346
28	6	23	1,213	1,21	1,208	1,041	1,394
28	6	24	1,259	1,256	1,252	1,078	1,453
28	6	25	1,314	1,31	1,306	1,119	1,522
28	6	26	1,38	1,376	1,37	1,166	1,614
28	6	27	1,472	1,466	1,458	1,226	1,754
28	6	28	1,633	1,623	1,605	1,319	2,028
28	9	1	0,525	0,516	0,52	0,326	0,687

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
28	9	2	0,605	0,6	0,602	0,448	0,746
28	9	3	0,656	0,653	0,653	0,513	0,786
28	9	4	0,696	0,693	0,693	0,565	0,819
28	9	5	0,73	0,727	0,727	0,603	0,848
28	9	6	0,76	0,757	0,756	0,638	0,876
28	9	7	0,787	0,785	0,784	0,669	0,899
28	9	8	0,813	0,81	0,81	0,695	0,924
28	9	9	0,837	0,835	0,834	0,722	0,95
28	9	10	0,861	0,859	0,858	0,747	0,972
28	9	11	0,883	0,881	0,881	0,77	0,994
28	9	12	0,906	0,904	0,903	0,791	1,016
28	9	13	0,928	0,926	0,925	0,812	1,041
28	9	14	0,95	0,948	0,948	0,831	1,062
28	9	15	0,972	0,97	0,969	0,854	1,087
28	9	16	0,994	0,993	0,992	0,876	1,111
28	9	17	1,017	1,015	1,014	0,897	1,137
28	9	18	1,04	1,038	1,037	0,92	1,16
28	9	19	1,064	1,062	1,061	0,943	1,187
28	9	20	1,089	1,087	1,086	0,966	1,216
28	9	21	1,116	1,114	1,112	0,99	1,247
28	9	22	1,145	1,143	1,142	1,014	1,282
28	9	23	1,177	1,175	1,173	1,042	1,32
28	9	24	1,214	1,211	1,21	1,071	1,363
28	9	25	1,256	1,253	1,25	1,104	1,42
28	9	26	1,307	1,304	1,301	1,14	1,486
28	9	27	1,378	1,374	1,368	1,19	1,594
28	9	28	1,503	1,497	1,481	1,261	1,822
28	12	1	0,588	0,582	0,586	0,407	0,733
28	12	2	0,661	0,657	0,66	0,519	0,782
28	12	3	0,707	0,705	0,706	0,584	0,82
28	12	4	0,743	0,741	0,742	0,626	0,85
28	12	5	0,773	0,771	0,77	0,662	0,876
28	12	6	0,798	0,797	0,797	0,691	0,897
28	12	7	0,822	0,82	0,821	0,721	0,921
28	12	8	0,844	0,843	0,842	0,744	0,943
28	12	9	0,865	0,864	0,863	0,766	0,961
28	12	10	0,885	0,884	0,884	0,786	0,98
28	12	11	0,904	0,903	0,903	0,806	0,997
28	12	12	0,924	0,922	0,922	0,827	1,018
28	12	13	0,943	0,941	0,941	0,846	1,038
28	12	14	0,962	0,96	0,959	0,864	1,059
28	12	15	0,981	0,98	0,979	0,882	1,081
28	12	16	1	0,999	0,997	0,901	1,099
28	12	17	1,019	1,018	1,018	0,92	1,119
28	12	18	1,04	1,038	1,038	0,939	1,14

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
28	12	19	1,061	1,059	1,059	0,958	1,165
28	12	20	1,082	1,08	1,079	0,976	1,19
28	12	21	1,105	1,103	1,102	0,998	1,216
28	12	22	1,129	1,128	1,126	1,019	1,243
28	12	23	1,156	1,155	1,152	1,042	1,277
28	12	24	1,187	1,185	1,183	1,067	1,316
28	12	25	1,222	1,22	1,217	1,095	1,361
28	12	26	1,267	1,264	1,261	1,126	1,421
28	12	27	1,326	1,323	1,317	1,164	1,507
28	12	28	1,431	1,426	1,412	1,223	1,687
30	2	1	0,056	0,04	0,029	0,001	0,145
30	2	2	0,098	0,081	0,069	0,01	0,216
30	2	3	0,138	0,122	0,11	0,027	0,278
30	2	4	0,179	0,162	0,152	0,047	0,336
30	2	5	0,221	0,204	0,194	0,071	0,395
30	2	6	0,262	0,246	0,236	0,097	0,452
30	2	7	0,304	0,288	0,279	0,125	0,507
30	2	8	0,347	0,331	0,322	0,154	0,56
30	2	9	0,39	0,375	0,366	0,185	0,614
30	2	10	0,435	0,419	0,411	0,218	0,667
30	2	11	0,48	0,464	0,457	0,252	0,721
30	2	12	0,525	0,51	0,503	0,287	0,775
30	2	13	0,572	0,557	0,55	0,323	0,832
30	2	14	0,621	0,606	0,599	0,363	0,889
30	2	15	0,671	0,656	0,649	0,403	0,947
30	2	16	0,723	0,708	0,702	0,444	1,01
30	2	17	0,777	0,762	0,755	0,488	1,074
30	2	18	0,833	0,818	0,812	0,532	1,141
30	2	19	0,892	0,877	0,871	0,58	1,211
30	2	20	0,954	0,939	0,933	0,631	1,285
30	2	21	1,02	1,005	0,999	0,683	1,363
30	2	22	1,091	1,076	1,069	0,739	1,451
30	2	23	1,168	1,152	1,145	0,799	1,545
30	2	24	1,253	1,237	1,228	0,866	1,65
30	2	25	1,346	1,33	1,32	0,937	1,766
30	2	26	1,455	1,437	1,427	1,018	1,91
30	2	27	1,583	1,564	1,553	1,113	2,081
30	2	28	1,744	1,722	1,707	1,225	2,306
30	2	29	1,967	1,94	1,917	1,366	2,646
30	2	30	2,368	2,324	2,277	1,577	3,344
30	3	1	0,183	0,162	0,153	0,029	0,352
30	3	2	0,262	0,246	0,239	0,092	0,437
30	3	3	0,323	0,31	0,305	0,146	0,501
30	3	4	0,376	0,365	0,36	0,196	0,557
30	3	5	0,424	0,414	0,41	0,241	0,609

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
30	3	6	0,469	0,459	0,456	0,283	0,656
30	3	7	0,511	0,502	0,499	0,324	0,699
30	3	8	0,552	0,543	0,54	0,362	0,744
30	3	9	0,592	0,584	0,581	0,4	0,785
30	3	10	0,631	0,623	0,62	0,438	0,826
30	3	11	0,67	0,662	0,659	0,474	0,869
30	3	12	0,708	0,701	0,698	0,509	0,909
30	3	13	0,746	0,739	0,737	0,544	0,951
30	3	14	0,785	0,778	0,776	0,58	0,992
30	3	15	0,824	0,817	0,814	0,615	1,037
30	3	16	0,863	0,856	0,853	0,648	1,08
30	3	17	0,904	0,897	0,894	0,685	1,124
30	3	18	0,945	0,938	0,935	0,723	1,171
30	3	19	0,988	0,981	0,977	0,761	1,22
30	3	20	1,033	1,026	1,022	0,8	1,27
30	3	21	1,081	1,073	1,07	0,84	1,328
30	3	22	1,131	1,124	1,119	0,882	1,386
30	3	23	1,185	1,177	1,173	0,928	1,451
30	3	24	1,244	1,236	1,232	0,976	1,521
30	3	25	1,309	1,301	1,297	1,029	1,601
30	3	26	1,384	1,375	1,37	1,088	1,693
30	3	27	1,472	1,462	1,454	1,153	1,809
30	3	28	1,582	1,57	1,561	1,229	1,964
30	3	29	1,733	1,719	1,704	1,33	2,19
30	3	30	1,999	1,976	1,945	1,47	2,659
30	6	1	0,41	0,397	0,4	0,197	0,588
30	6	2	0,497	0,489	0,49	0,319	0,657
30	6	3	0,555	0,549	0,55	0,393	0,706
30	6	4	0,601	0,596	0,596	0,448	0,745
30	6	5	0,64	0,636	0,636	0,492	0,78
30	6	6	0,675	0,671	0,671	0,531	0,813
30	6	7	0,708	0,704	0,703	0,566	0,844
30	6	8	0,738	0,734	0,734	0,6	0,873
30	6	9	0,767	0,763	0,763	0,63	0,902
30	6	10	0,794	0,791	0,79	0,658	0,928
30	6	11	0,82	0,817	0,816	0,685	0,954
30	6	12	0,846	0,843	0,842	0,71	0,982
30	6	13	0,872	0,869	0,869	0,736	1,008
30	6	14	0,898	0,895	0,894	0,759	1,036

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
30	6	15	0,923	0,921	0,92	0,785	1,061
30	6	16	0,949	0,947	0,946	0,81	1,089
30	6	17	0,975	0,973	0,972	0,836	1,117
30	6	18	1,002	0,999	0,998	0,861	1,145
30	6	19	1,029	1,027	1,026	0,886	1,177
30	6	20	1,058	1,055	1,054	0,913	1,209
30	6	21	1,088	1,085	1,082	0,937	1,245
30	6	22	1,119	1,116	1,114	0,965	1,279
30	6	23	1,152	1,149	1,147	0,995	1,315
30	6	24	1,189	1,186	1,184	1,027	1,36
30	6	25	1,23	1,226	1,224	1,059	1,408
30	6	26	1,275	1,271	1,269	1,095	1,465
30	6	27	1,328	1,324	1,32	1,135	1,533
30	6	28	1,393	1,389	1,383	1,184	1,624
30	6	29	1,484	1,478	1,468	1,243	1,758
30	6	30	1,645	1,635	1,615	1,331	2,041
30	9	1	0,519	0,51	0,515	0,32	0,677
30	9	2	0,598	0,593	0,594	0,441	0,732
30	9	3	0,649	0,645	0,646	0,51	0,774
30	9	4	0,688	0,685	0,685	0,56	0,809
30	9	5	0,721	0,718	0,718	0,599	0,835
30	9	6	0,75	0,747	0,747	0,632	0,861
30	9	7	0,776	0,774	0,774	0,661	0,884
30	9	8	0,801	0,799	0,799	0,688	0,908
30	9	9	0,824	0,822	0,822	0,711	0,931
30	9	10	0,846	0,844	0,844	0,736	0,953
30	9	11	0,868	0,866	0,866	0,76	0,974
30	9	12	0,889	0,887	0,886	0,782	0,997
30	9	13	0,91	0,908	0,907	0,801	1,017
30	9	14	0,93	0,928	0,928	0,821	1,037
30	9	15	0,95	0,949	0,949	0,84	1,058
30	9	16	0,971	0,969	0,969	0,861	1,08
30	9	17	0,992	0,99	0,99	0,882	1,102
30	9	18	1,013	1,011	1,011	0,902	1,123
30	9	19	1,034	1,033	1,032	0,92	1,147
30	9	20	1,057	1,055	1,055	0,941	1,172
30	9	21	1,08	1,078	1,078	0,961	1,199
30	9	22	1,105	1,103	1,102	0,985	1,228
30	9	23	1,131	1,129	1,128	1,007	1,256

Ns	Nr	i	QM	Mean	Med	c2,5%	c97,5%
30	9	24	1,16	1,158	1,156	1,033	1,292
30	9	25	1,191	1,189	1,187	1,06	1,33
30	9	26	1,226	1,224	1,221	1,087	1,371
30	9	27	1,267	1,265	1,261	1,12	1,428
30	9	28	1,319	1,316	1,311	1,156	1,501
30	9	29	1,389	1,385	1,379	1,204	1,608
30	9	30	1,513	1,506	1,491	1,27	1,815
30	12	1	0,583	0,576	0,582	0,4	0,726
30	12	2	0,654	0,651	0,652	0,513	0,773
30	12	3	0,7	0,697	0,698	0,576	0,809
30	12	4	0,734	0,732	0,733	0,62	0,839
30	12	5	0,763	0,762	0,762	0,656	0,863
30	12	6	0,789	0,787	0,788	0,687	0,886
30	12	7	0,812	0,81	0,811	0,709	0,908
30	12	8	0,833	0,832	0,832	0,734	0,927
30	12	9	0,854	0,852	0,853	0,756	0,946
30	12	10	0,873	0,872	0,872	0,777	0,964
30	12	11	0,892	0,89	0,891	0,797	0,984
30	12	12	0,91	0,908	0,909	0,814	1,001
30	12	13	0,927	0,926	0,926	0,832	1,021
30	12	14	0,945	0,943	0,943	0,851	1,038
30	12	15	0,962	0,961	0,96	0,869	1,056
30	12	16	0,98	0,978	0,978	0,888	1,074
30	12	17	0,997	0,996	0,996	0,903	1,094
30	12	18	1,015	1,014	1,014	0,921	1,111
30	12	19	1,033	1,032	1,031	0,936	1,131
30	12	20	1,053	1,051	1,051	0,953	1,152
30	12	21	1,073	1,071	1,07	0,972	1,175
30	12	22	1,093	1,092	1,091	0,99	1,2
30	12	23	1,116	1,115	1,113	1,011	1,228
30	12	24	1,14	1,138	1,137	1,031	1,254
30	12	25	1,166	1,165	1,163	1,052	1,286
30	12	26	1,196	1,194	1,192	1,077	1,325
30	12	27	1,231	1,229	1,226	1,105	1,37
30	12	28	1,273	1,271	1,268	1,135	1,429
30	12	29	1,333	1,33	1,324	1,173	1,516
30	12	30	1,434	1,429	1,418	1,232	1,691

Table A5: Values of scatter of the estimates of a standard deviation from repeated series
 N_r is the number of repetitions, N_s is the number of series, s_s is the standard deviations of estimates of s/σ ,

IC- and IC+ are the lower and upper limits of the 95% interval of confidence of estimates of s/σ

(Differences between the regression method and the usual method are lower than the uncertainties of determinations)

Nr	Ns	Regression method			Usual method		
		s_s	IC-	IC+	s_s	IC-	IC+
2	5	0,357	0,353	1,725	0,353	0,357	1,720
2	6	0,329	0,399	1,676	0,324	0,405	1,663
2	7	0,309	0,431	1,630	0,304	0,437	1,621
2	8	0,291	0,460	1,584	0,287	0,464	1,573
2	9	0,276	0,484	1,559	0,272	0,487	1,551
2	10	0,265	0,509	1,535	0,262	0,512	1,525
2	13	0,233	0,558	1,465	0,230	0,562	1,460
2	16	0,214	0,592	1,430	0,211	0,597	1,421
2	20	0,190	0,633	1,373	0,188	0,635	1,371
2	25	0,172	0,671	1,342	0,170	0,673	1,339
2	32	0,154	0,704	1,307	0,152	0,707	1,302
2	40	0,137	0,737	1,274	0,136	0,740	1,272
2	50	0,122	0,763	1,241	0,121	0,766	1,238
2	63	0,110	0,789	1,217	0,109	0,790	1,218
3	5	0,318	0,411	1,647	0,314	0,416	1,633
3	6	0,294	0,453	1,596	0,291	0,456	1,589
3	7	0,276	0,478	1,550	0,274	0,482	1,543
3	8	0,262	0,503	1,526	0,259	0,506	1,519
3	9	0,251	0,523	1,503	0,249	0,526	1,494
3	10	0,238	0,546	1,477	0,236	0,550	1,472
3	13	0,213	0,596	1,426	0,211	0,598	1,421
3	16	0,192	0,630	1,381	0,191	0,634	1,378
3	20	0,174	0,664	1,345	0,173	0,667	1,339
3	25	0,157	0,696	1,307	0,156	0,697	1,306
3	32	0,140	0,730	1,278	0,139	0,731	1,276
3	40	0,125	0,756	1,248	0,125	0,757	1,247
3	50	0,112	0,782	1,221	0,111	0,783	1,221
3	63	0,101	0,803	1,198	0,100	0,805	1,197
4	5	0,292	0,455	1,587	0,289	0,461	1,584
4	6	0,274	0,483	1,553	0,272	0,489	1,551
4	7	0,261	0,507	1,519	0,259	0,510	1,519
4	8	0,247	0,533	1,488	0,246	0,536	1,489
4	9	0,236	0,548	1,467	0,235	0,552	1,466
4	10	0,227	0,566	1,450	0,226	0,568	1,451
4	13	0,202	0,610	1,397	0,201	0,614	1,393
4	16	0,185	0,645	1,365	0,185	0,648	1,365
4	20	0,167	0,678	1,331	0,166	0,680	1,329
4	25	0,150	0,708	1,296	0,149	0,710	1,294
4	32	0,135	0,740	1,264	0,134	0,742	1,264
4	40	0,120	0,766	1,239	0,120	0,768	1,236

Nr	Ns	Regression method			Usual method		
		s_s	IC-	IC+	s_s	IC-	IC+
4	50	0,108	0,789	1,215	0,108	0,790	1,213
4	63	0,096	0,813	1,189	0,096	0,813	1,188
5	5	0,277	0,482	1,554	0,274	0,488	1,551
5	6	0,261	0,507	1,526	0,259	0,512	1,525
5	7	0,247	0,533	1,491	0,246	0,537	1,491
5	8	0,237	0,548	1,472	0,236	0,553	1,469
5	9	0,225	0,570	1,447	0,224	0,574	1,443
5	10	0,218	0,586	1,441	0,217	0,588	1,441
5	13	0,196	0,623	1,391	0,195	0,627	1,388
5	16	0,178	0,656	1,352	0,178	0,658	1,350
5	20	0,163	0,682	1,319	0,163	0,683	1,318
5	25	0,147	0,715	1,288	0,146	0,717	1,287
5	32	0,130	0,747	1,255	0,129	0,747	1,254
5	40	0,117	0,773	1,229	0,116	0,774	1,228
5	50	0,106	0,797	1,209	0,105	0,797	1,207
5	63	0,095	0,816	1,187	0,094	0,816	1,185
6	5	0,257	0,514	1,516	0,255	0,518	1,512
6	6	0,249	0,529	1,503	0,247	0,533	1,501
6	7	0,238	0,550	1,475	0,236	0,552	1,471
6	8	0,229	0,568	1,456	0,227	0,571	1,454
6	9	0,219	0,581	1,437	0,218	0,585	1,434
6	10	0,210	0,597	1,417	0,209	0,599	1,414
6	13	0,190	0,639	1,375	0,189	0,642	1,376
6	16	0,175	0,663	1,344	0,174	0,664	1,342
6	20	0,159	0,693	1,317	0,158	0,694	1,315
6	25	0,144	0,723	1,286	0,143	0,723	1,284
6	32	0,129	0,750	1,253	0,128	0,751	1,252
6	40	0,115	0,777	1,226	0,114	0,778	1,224
6	50	0,104	0,798	1,206	0,104	0,798	1,203
6	63	0,094	0,817	1,183	0,093	0,817	1,182
7	5	0,244	0,540	1,488	0,242	0,543	1,485
7	6	0,236	0,548	1,472	0,234	0,551	1,468
7	7	0,228	0,565	1,459	0,227	0,568	1,456
7	8	0,220	0,586	1,440	0,218	0,588	1,438
7	9	0,212	0,598	1,420	0,211	0,600	1,418
7	10	0,203	0,608	1,401	0,202	0,610	1,399
7	13	0,185	0,646	1,367	0,184	0,647	1,366
7	16	0,171	0,668	1,337	0,170	0,671	1,336
7	20	0,155	0,701	1,309	0,155	0,702	1,308
7	25	0,141	0,725	1,277	0,141	0,726	1,277

Nr	Ns	Regression method			Usual method		
		s _s	IC-	IC+	s _s	IC-	IC+
7	32	0,126	0,754	1,249	0,126	0,755	1,249
7	40	0,114	0,777	1,223	0,114	0,778	1,222
7	50	0,102	0,801	1,200	0,102	0,801	1,200
8	5	0,232	0,557	1,463	0,231	0,559	1,462
8	6	0,226	0,573	1,453	0,224	0,576	1,451
8	7	0,218	0,584	1,433	0,217	0,587	1,431
8	8	0,213	0,595	1,428	0,212	0,598	1,425
8	9	0,205	0,607	1,407	0,204	0,610	1,406
8	10	0,198	0,626	1,400	0,197	0,630	1,399
8	13	0,182	0,650	1,361	0,181	0,654	1,362
8	16	0,167	0,679	1,332	0,166	0,680	1,331
8	20	0,152	0,705	1,304	0,152	0,707	1,304
8	25	0,140	0,727	1,273	0,140	0,730	1,273
8	32	0,125	0,759	1,248	0,124	0,760	1,248
8	40	0,112	0,781	1,218	0,112	0,782	1,218
9	5	0,221	0,581	1,443	0,220	0,582	1,439
9	6	0,214	0,588	1,425	0,213	0,589	1,422
9	7	0,209	0,604	1,415	0,208	0,605	1,412
9	8	0,204	0,614	1,409	0,203	0,615	1,406
9	9	0,198	0,622	1,401	0,197	0,624	1,398
9	10	0,193	0,635	1,392	0,192	0,637	1,387
9	13	0,177	0,662	1,351	0,176	0,664	1,349
9	16	0,164	0,684	1,328	0,163	0,684	1,327
9	20	0,151	0,710	1,301	0,151	0,711	1,301
9	25	0,138	0,733	1,271	0,138	0,734	1,271
9	32	0,123	0,763	1,245	0,123	0,763	1,246
9	40	0,112	0,782	1,221	0,112	0,782	1,221
10	5	0,212	0,593	1,420	0,211	0,597	1,421
10	6	0,205	0,603	1,402	0,205	0,608	1,403
10	7	0,203	0,612	1,404	0,202	0,615	1,405
10	8	0,197	0,623	1,394	0,196	0,625	1,395
10	9	0,192	0,634	1,382	0,191	0,637	1,382
10	10	0,186	0,638	1,363	0,186	0,641	1,363
10	13	0,174	0,666	1,344	0,173	0,669	1,345
10	16	0,163	0,687	1,323	0,162	0,690	1,324
10	20	0,148	0,711	1,292	0,148	0,713	1,294
10	25	0,136	0,736	1,266	0,136	0,737	1,267
10	32	0,122	0,760	1,236	0,121	0,762	1,238
11	5	0,203	0,607	1,401	0,202	0,611	1,402
11	6	0,199	0,616	1,394	0,199	0,619	1,395
11	7	0,194	0,627	1,385	0,194	0,631	1,386
11	8	0,190	0,635	1,378	0,190	0,639	1,378
11	9	0,185	0,643	1,363	0,185	0,646	1,364
11	10	0,183	0,649	1,364	0,182	0,653	1,364

Nr	Ns	Regression method			Usual method		
		s _s	IC-	IC+	s _s	IC-	IC+
11	13	0,170	0,674	1,336	0,169	0,676	1,337
11	16	0,160	0,689	1,312	0,159	0,691	1,313
11	20	0,147	0,715	1,287	0,147	0,716	1,289
11	25	0,135	0,736	1,264	0,134	0,738	1,263
11	32	0,121	0,761	1,237	0,121	0,762	1,238